

CSE 4214 :: Lab 1

Issued October 7, 2008; due October 21, 2008

This lab will familiarize you with matched-filter detection of signals, and will give you experience in running numerical simulations of communication systems in MATLAB.

Although you are allowed (and encouraged) to consult with other students in performing this lab, your work must be submitted individually. Group submissions are not allowed.

Section 1. Signals and matched filters.

Here are three signal sets:

1	$s_0(t) = \begin{cases} 1, & 0 \leq t < T/2 \\ -1, & T/2 \leq t \leq T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = -s_0(t)$
2	$s_0(t) = \begin{cases} \sin 2\pi t / T, & 0 \leq t \leq T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = -s_0(t)$
3	$s_0(t) = \begin{cases} \sin 2\pi t / T, & 0 \leq t \leq T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = \begin{cases} \cos 2\pi t / T, & 0 \leq t \leq T \\ 0, & t < 0, t > T \end{cases}$

For each signal set, do the following:

1. Sketch (or plot using MATLAB) the signals $s_0(t)$ and $s_1(t)$;
2. Derive and sketch/plot the impulse response $h(t)$ of the matched filter (matched to $s_0(t)$);
3. Calculate the probability of error for each signal set assuming that $T = 1$; and
4. Again assuming $T=1$, find the value of N_0 such that the probability of error is 0.25, and the value of N_0 such that the probability of error is 0.01.

Section 2. Simulating the matched filter.

In this part you will implement the matched filters you derived in part 1. Assume $T = 1$.

Clearly, the signals operate in continuous time, and we proved the matched filter was optimal for continuous-time signals. However, MATLAB requires you to work in discrete time. Thus, you will be sampling the signal, and feeding the sampled signal to a discrete-time filter, which is the sampled version of the matched filter.

If the simulated system is a sampled version of the real system, what should we use as the sampling frequency? Nyquist sampling theory suggests that the sampling frequency should be twice the bandwidth of the signal. However, our signals are bounded in time between $t=0$ and

$t=T$; furthermore, it is known that signals that are limited in time have no finite bandwidth. Thus, the sampling frequency should (ideally) be infinite. In practice, we may select the sampling frequency to be large enough not to cause errors.

For your simulations, try sampling frequencies of $5/T$, $10/T$, and $20/T$. This corresponds, respectively, to 5, 10, and 20 samples per signal.

Note, caution is to be exercised when generating the sampled additive white Gaussian noise process at the input to the matched filter. Its variance must be carefully selected to obtain the correct variance at the filter output. This is left as a problem for the student.

For all signal sets and these three sampling frequencies, do the following:

1. In part 4 of section 1, you found two values of N_0 for different probabilities of error. Between these two values of N_0 , find three equally spaced additional values (e.g., if the lower value is 1, and the upper value is 5, the intermediate values would be 2, 3, and 4). You will end up with a total of five values (in the example, you have 1, 2, 3, 4, 5).
2. For each of the five values, generate 1000 bits of binary data at random (where the digits 0 and 1 have equal probability). For each bit, generate the (sampled) signal $s_0(t)$ or $s_1(t)$, as appropriate; add Gaussian noise; pass the noisy signal through the (discrete-time) matched filter; and (based on the matched filter output) decide whether the bit is 0 or 1. Calculate the probability of error from the simulation (e.g., if the decision is correct 800 times and incorrect 200 times, the error rate is 0.2).
3. Using the MATLAB plotting features, plot the simulated error rate versus N_0 on a log-log scale. Generate one plot for each signal set, and plot the results for the three sampling frequencies on the same plot. Note that your results should match reasonably well with the values you found in part 4 of section 1, especially for the highest sampling frequency.

Deliverables

Your deliverables for this lab are:

- Answers for the four parts in section 1;
- Your MATLAB code for section 2; and
- Plots for the three signal sets, from part 3 of section 2.