CSE 4214 :: Lab 4

Issued February 9, 2009; Due February 19, 2009.
Will be accepted without penalty until February 26, 2009.

This lab introduces you to the Hamming code.

Although you are allowed (and encouraged) to consult with other students in performing this lab, your work must be submitted individually. Group submissions are not allowed.

Section 1. Hamming code generator and parity check matrices

Consider the following generator matrix for a Hamming code:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1. Find the corresponding parity check matrix H.
2. Show that the minimum Hamming distance of the code is 3.
3. Calculate the syndrome for all seven single-error patterns.
4. Describe the algorithm for correcting up to a single error. Under what circumstances does this decoding algorithm fail to correct errors?
5. Describe the algorithm for detecting up to two errors. Under what circumstances does this detection algorithm fail to detect errors?

Section 2. Simulation.

In this part you will simulate both the error correction algorithms from Part 1.

For these simulations, the transmitted signal will be generated by producing a random row vector b of four information bits, where each bit in the vector can be 0 or 1 with equal probability, and each bit is selected independently; and generating a codeword by multiplying b by the generator matrix G (mod 2). The received signal is generated as follows: each bit in the transmitted signal is either flipped (from 0 to 1 or vice versa) with probability p, or left alone with probability 1-p, and each bit is treated independently.

For the simulations, use the following values of p: 0.2, 0.1, 0.07, 0.05, 0.035, 0.02, 0.01, 0.007.

Error correction: Implement the algorithm you obtained in question 4 of part 1. For each value of p in the above list, simulate 1000 codewords. For the codeword you obtain as the output of the error correction algorithm, count the number of errors in only the information bits, and plot the bit error rate with respect to p on a log-log scale.

Error detection: Implement the algorithm you obtained in question 5 of part 1. For each value of p in the above list, simulate 1000 codewords. For each codeword, if the algorithm detects errors, simulate the re-sending of the codeword. The codeword is re-sent until either your algorithm detects no errors, or until the codeword has been re-sent 10 times with errors. (These extra transmissions do NOT count as part of the 1000 codewords!) Once the algorithm terminates, check the received codeword and determine whether there are any undetected errors only in the information bits. Plot the rate of undetected errors with respect to p on a log-log scale.

Deliverables

Your deliverables for this lab are:

- Answers for the four parts in section 1;
- Your MATLAB code for section 2; and
- Plots from section 2.