CSE 4214 :: Problem Set 2

1. Consider the following signal sets:

#1	$s_0(t) = \begin{cases} 1, & 0 \le t < T/2 \\ -1, & T/2 \le t \le T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = -s_0(t)$
#2	$s_0(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = 0$
#3	$s_0(t) = \begin{cases} \sin 2\pi t/T, & 0 \le t \le T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = \begin{cases} \cos 2\pi t/T, & 0 \le t \le T \\ 0, & t < 0, t > T \end{cases}$

   a. For each signal set, sketch the data sequence: 10101011
   b. What is the average energy per bit in each signal set?

2. For signal set #1 in the above table:

   a. Determine the impulse response of the filter matched to $s_0(t)$, and sketch it as a function of time.
   b. Plot the matched filter output as a function of time.
   c. What is the peak value of the output, and when does it occur?

3. For signal set #3 in the above table:

   a. Show that the two signals, $s_0(t)$ and $s_1(t)$, are orthogonal.
   b. Determine the impulse response of the filter matched to $s_0(t)$, and the filter matched to $s_1(t)$.
   c. Use your result from part a to show that, if $s_0(t)$ is applied to the filter matched to $s_1(t)$, the output is zero, and vice versa.

4. For signal set #2 in the above table, find the expression for the probability of detection error, assuming that the signal is received in the presence of additive white Gaussian noise with power spectral density $N_0/2$.

5. Repeat question 4, but letting $s_1(t) = -s_0(t)$. Between the original signal set #2, and this signal set, which is more energy efficient? Explain.

6. Suppose at the output of the matched filter, the amplitude assuming 0 was sent is +A, the amplitude assuming 1 was sent is –A, and the noise variance is 0.0001.

   a. Find the regions where the optimal decision is 0 and 1, and the boundary between the decision regions.
   b. Calculate the value of A such that the probability of error is 10^{-6}. (Hint: $\text{erfc}(3.36)/2 = 10^{-6}$).