

## Combinational Logic: Design of a 1-bit adder (2)

Step 2: Derive the Boolean expression for each output from the truth table

| INPUTS |  |  | OUTPUTS |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{c}$ <br> (CarryIn) | CarryOut | Sum |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |



Sum $=\bar{a} \bar{b} c+\bar{a} b \bar{c}+a \bar{b} \bar{c}+a b c$
Carryout $=\bar{a} b c+a \bar{b} c+a b \bar{c}+a b c$

## Combinational Logic: Design of a 1-bit adder (3)

Step 3: Simplify the Boolean expression

$$
\text { Carryout }=\bar{a} b c+a \bar{b} c+a b \bar{c}+a b c=b c+a c+a b
$$

Step 4: Implement the simplified Boolean expression using OR, AND, and NOT gates


Activity: Implement the hardware for the Sum output of the 1-bit adder

## Agenda for Today

$\square$ Introduction to Hardware - Logic Design

Patterson: Appendix C

## 1-bit adder

- Recall the digital circuit of a 1-bit adder
- We will enhance the 1-bit adder to develop a prototype ALU for MIPS


Digital Circuit of a 1-bit adder (CarryOut only)


Schematic of a 1-bit adder

## 1-bit ALU with AND, OR, and Addition

- The 1-bit adder is supplemented with AND and OR gates
- A multiplexer controls which gate is connected to the output


1-bit ALU with AND, OR, and Addition capability

| ALU Control Lines |  | Result |
| :---: | :---: | :---: |
| Carry In | Operation |  |
| $\mathbf{0}$ | $\mathbf{0}=(\mathbf{0 0})_{\mathrm{two}}$ | AND |
| $\mathbf{0}$ | $\mathbf{1}=(\mathbf{0 1})_{\mathrm{two}}$ | OR |
| $\mathbf{0}$ | $\mathbf{2}=(\mathbf{1 0})_{\mathbf{t w o}}$ | add |



## 32-bit ALU w/ AND, OR, and ADD

- The 1-bit ALU can be cascaded together to form a 32 bit ALU
- Which operation is performed is controlled by the Operation bus

| ALU Control Lines |  | Result |
| :---: | :---: | :---: |
| Carry In | Operation |  |
| $\mathbf{0}$ | $\mathbf{0}=(\mathbf{0 0})_{\text {two }}$ | AND |
| $\mathbf{0}$ | $\mathbf{1}=(\mathbf{( 0 1})_{\text {two }}$ | OR |
| $\mathbf{0}$ | $\mathbf{2}=(\mathbf{1 0})_{\mathrm{two}}$ | add |

- The designed 32-bit ALU is still missing the subtraction, slt (set if less than), and conditional branch operations



## 1-bit ALU with AND, OR, Addition, and Subtraction

- Recall that subtraction is performed using 2's complement arithmetic
- We calculate the 2's compliment of the sub-operand and add to the first operand


| ALU Control Lines |  |  | Result |
| :---: | :---: | :--- | :---: |
| Binvert | Carry In | Operation |  |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}=(\mathbf{0 0})_{\mathrm{two}}$ | AND |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}=(\mathbf{0 1})_{\mathrm{two}}$ | OR |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{2}=(\mathbf{1 0})_{\mathrm{two}}$ | add |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}=(\mathbf{1 0})_{\mathrm{two}}$ | sub |

1-bit ALU with AND, OR, Addition, and Subtraction capability

## 1-bit ALU with AND, OR, Add, Sub, and SLT (1)



- Since we need to perform one more operation, we increase the number of inputs at the multiplexer by 1 and label the new input as Less
- SLT operation: if $(\mathrm{a}<\mathrm{b})$, set Less to 1 $=>$ if $(a-b)<0$, set Less to 1
- SLT operation can therefore be expressed in terms of a subtraction between the two operands.
- If the result of subtraction is negative, set Less to 1 .
- How do we determine if the result is negative?


1-bit ALU with AND, OR, Add, Sub, and SLT capability

## 1-bit ALU with AND, OR, Add, Sub, and SLT (2) <br> 

- Use the sign bit obtained from the 1-bit ALU at the MSB position to indicate the result of SLT.


1-bit ALU of MSB (bit 31) with AND, OR, Add, Sub, and SLT capability

## 1-bit ALU with AND, OR, Add, Sub, and SLT (3)

- In fact we also include the overflow circuit within the 1-bit ALU at the MSB


1-bit ALU (4)



## 32-bit ALU

## w/ And, OR, Add, Subtract, and SLT

- The 1-bit ALU's can be cascaded together to form a 32 bit ALU
- Operations are controlled by the Operation bus

| ALU Control Lines |  |  | Result |
| :---: | :---: | :---: | :---: |
| Binvert | Carry In | Operation |  |
| 0 | 0 | $0=(00)_{\text {two }}$ | AND (a•b) |
| 0 | 0 | $1=(01)_{\text {two }}$ | OR (a+b) |
| 0 | 0 | 2 = (10) ${ }_{\text {two }}$ | Add sum( $\mathrm{a}, \mathrm{b}$ ) |
| 1 | 1 | 2 = (11) $)_{\text {two }}$ | Subtract (a-b) |
| 1 | 1 | 3 = $(11)_{\text {two }}$ | SLT <br> Set Result0 if ( $\mathrm{a}<\mathrm{b}$ ) |

- Note that Binvert is always the same as Carry In
- To test equality between $a$ and $b$, subtract $b$ from a and check if the result is 0 .



# 32-bit ALU w/ And, OR, Add, Subtract, SLT, and Equality Test 

| ALU Control Lines |  |  | Result |
| :---: | :---: | :--- | :---: |
| Binvert | Carry In | Operation |  |
| 0 | 0 | $0=(00)_{\mathrm{two}}$ | AND (a•b) |
| 0 | 0 | $1=(01)_{\mathrm{two}}$ | OR (a+b) |
| 0 | 0 | $2=(10)_{\mathrm{two}}$ | Add sum(a,b) |
| 1 | 1 | $2=(10)_{\mathrm{two}}$ | Subtract (a - b) |$|$| SLT |
| :---: |
| 1 |



## 32-bit ALU w/ And, OR, Add,

Subtract, SLT, and Equality Test


