Concurrent Red-Black Trees

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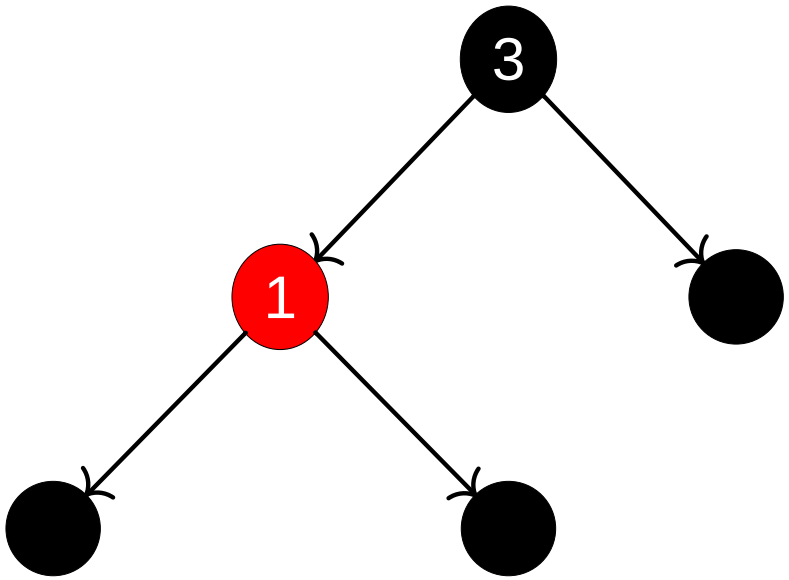
Red-Black Tree

A red-black tree is a binary search tree the nodes of which are coloured either red or black and

- the root is black,
- every leaf is black,
- if a node is red, then both its children are black,
- for every node, every path from that node to a leaf contains the same number of black nodes.

[Bayer, 1972] and [Guibas and Sedgewick, 1978]



Red-Black Tree

Theorem

A red-black tree with n internal nodes has height at most $2\log_2(n+1)$.

Corollary

The SET *operations* ADD *and* CONTAINS *can be implemented in* $O(\log_2(n))$.

Java Standard Library

The class java.util.TreeSet

```
1 class TreeSet<T>
2 {
3     boolean add(T element)
4     boolean contains(T element)
5     ...
6 }
```

has been implemented by means of a red-black tree.

This implementation is not synchronized.

Concurrent Operations on a Red-Black Tree

```
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2 add(1);
3 (add(2) || print(contains(1)))
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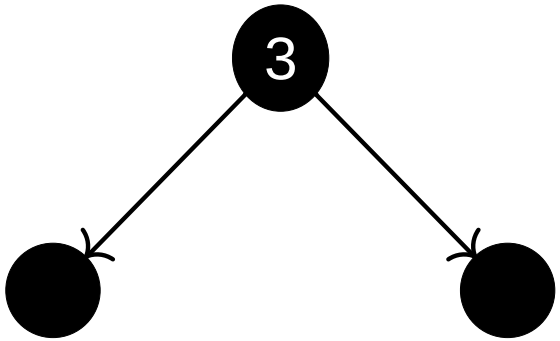

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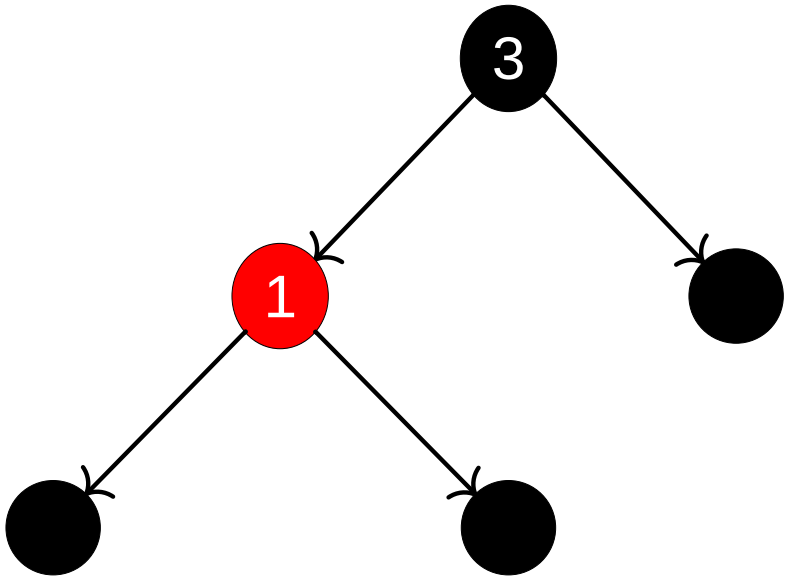

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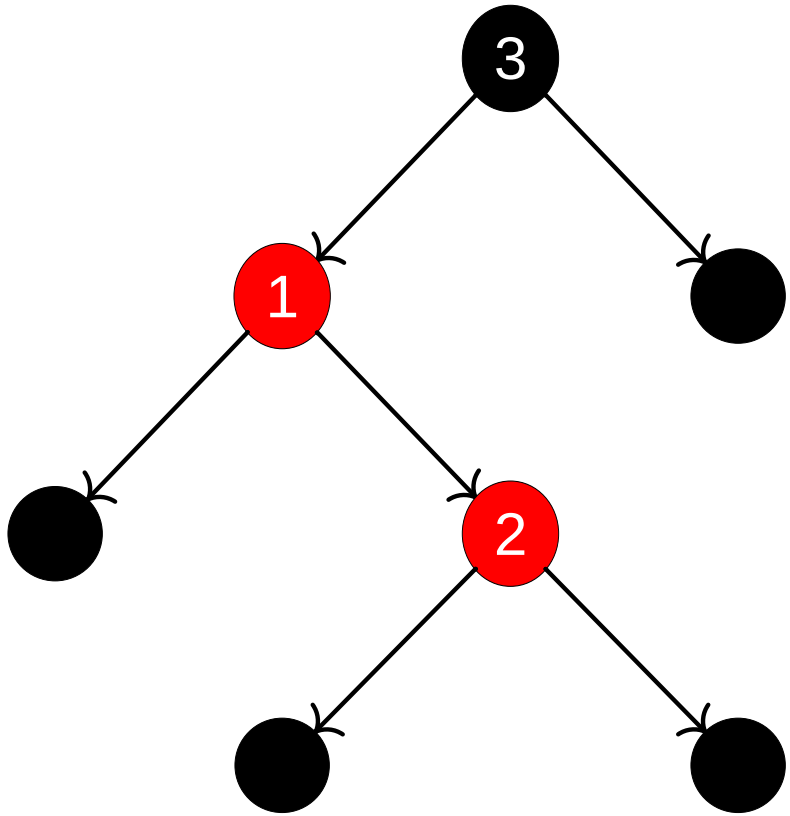

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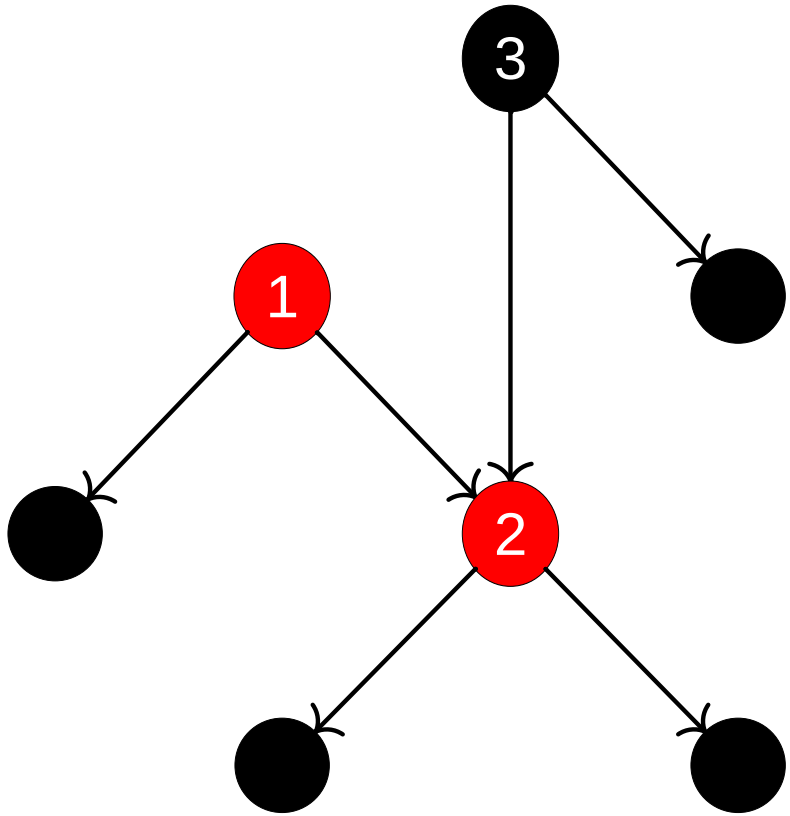

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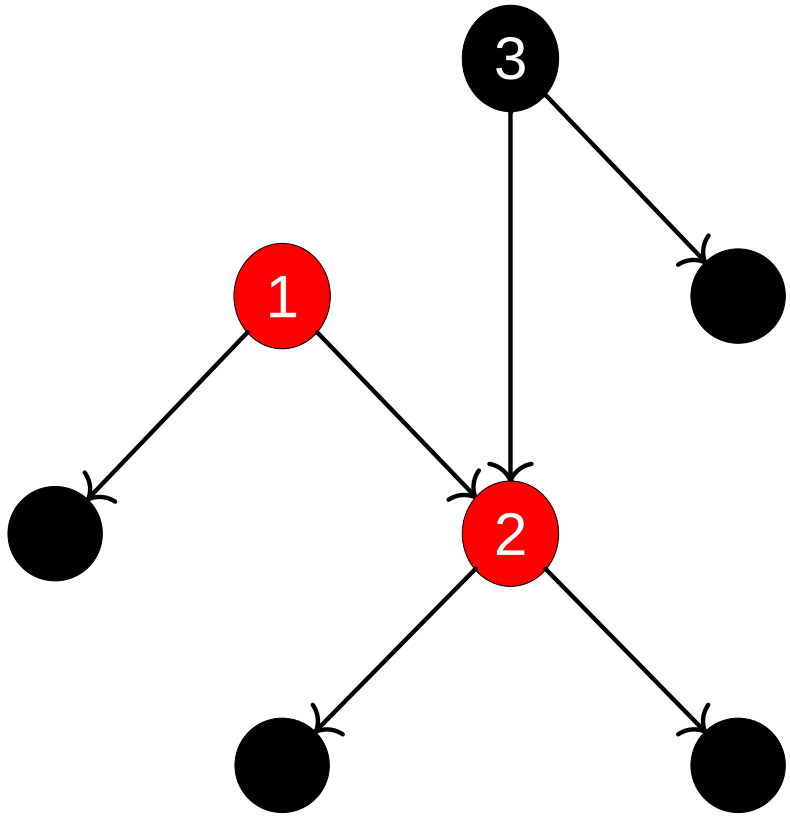


Concurrent Red-Black Trees

With the arrival of multicore machines, implementations of data structures such as Set should support concurrency.

In the remainder of this talk, three concurrent implementations of red-black trees are presented.

The Monitor Solution

```
1 RedBlackTree : monitor
2 begin
3   procedure add(element : int,
4                 result added : boolean)
5   procedure contains(element : int,
6                      result contains : boolean)
7 end
```

The Readers-Writers Solution

The processes of the first class, named *writers*, must have exclusive access, and the processes of the second class, the *readers*, may share the resource with an unlimited number of other readers.

The Readers-Writers Solution

The processes of the first class, those that call add, must have exclusive access, and the processes of the second class, those that call contains, may share the red-black tree with an unlimited number of such processes.

The Readers-Writers Solution

```
1 contains(element : int) : boolean
2 [manipulate shared variables, block/unblock]
3 manipulate red-black tree
4 [manipulate shared variables, unblock]
```

My Paper

Carla Schlatter Ellis. Concurrent Search and Insertion in AVL Trees. *IEEE Transactions on Computers*, 29(9):811–817, September 1980.

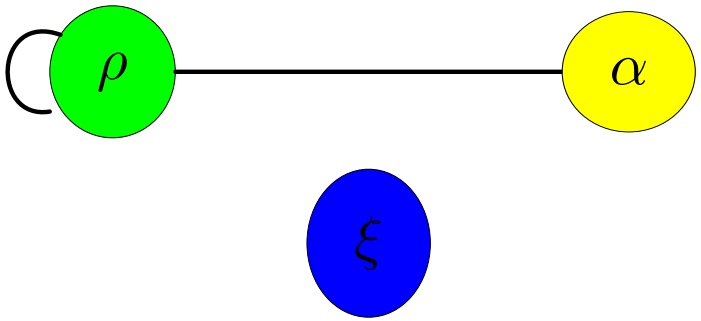
Carla Schlatter Ellis. *The Design and Evaluation of Algorithms for Parallel Processing*. PhD thesis, University of Washington, Seattle, 1979.

The Main Idea

Processes lock the nodes of the red-black tree in three different ways:

- ρ-lock: lock to read
- α-lock: lock to exclude writers
- ξ-lock: exclusive lock

Although a node can be locked by multiple processes, there are some restrictions.



Example Revisited

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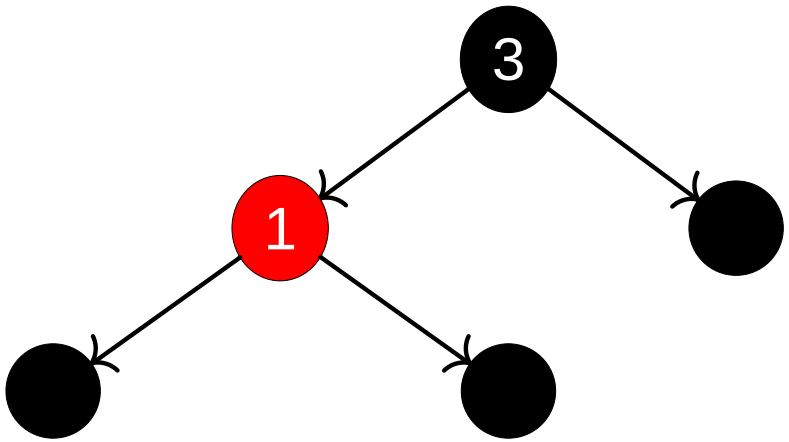

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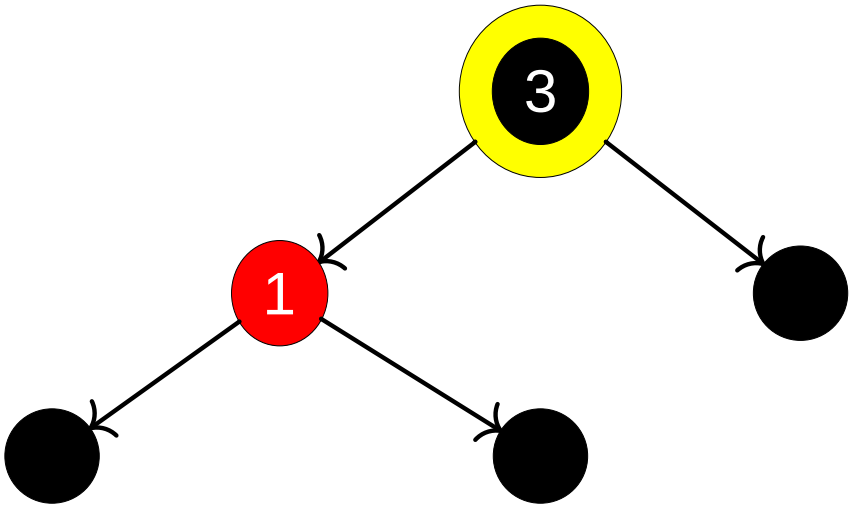

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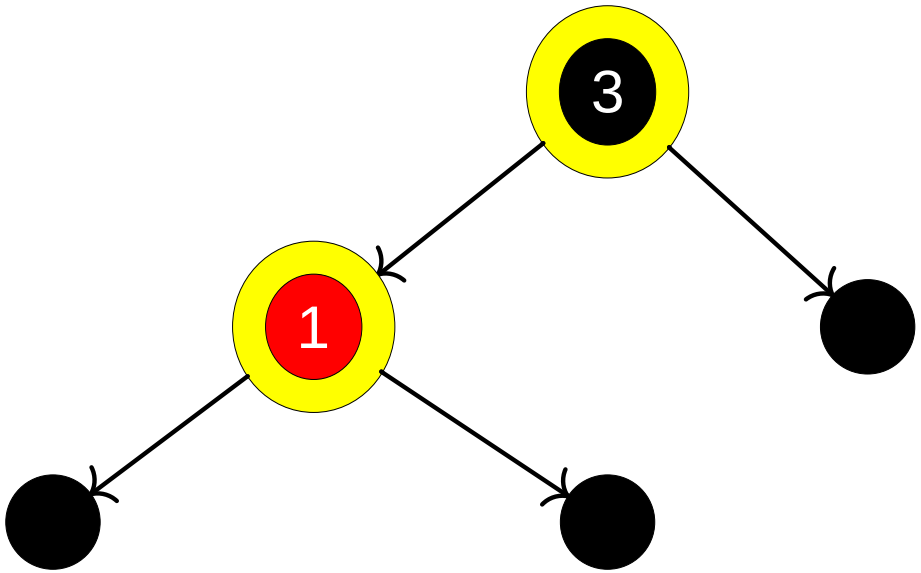

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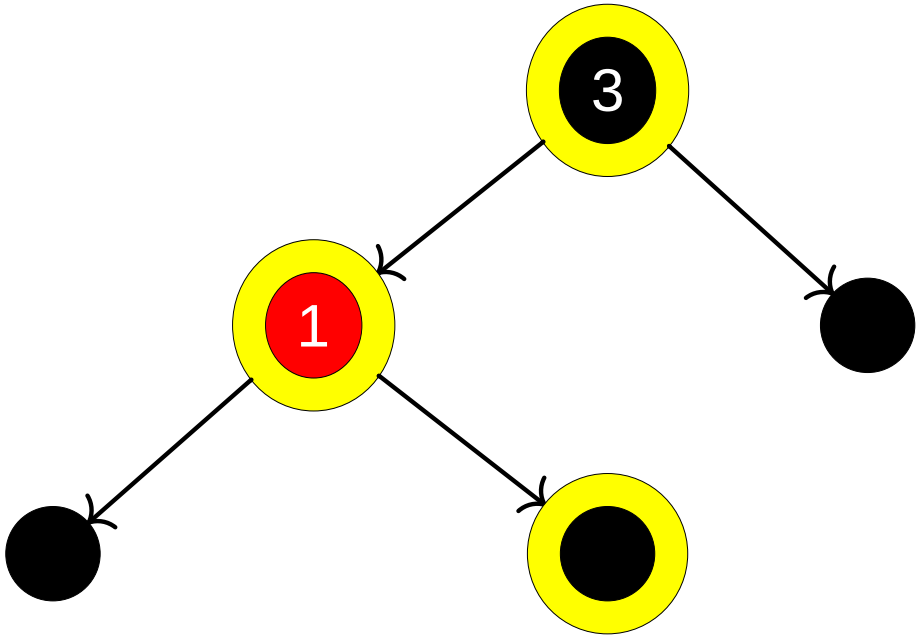

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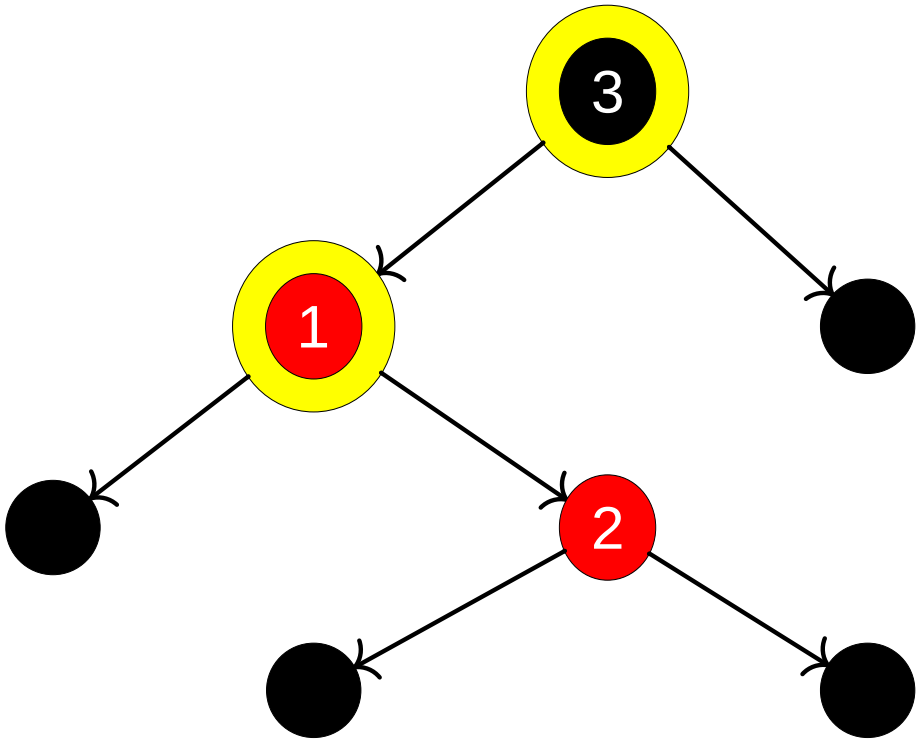

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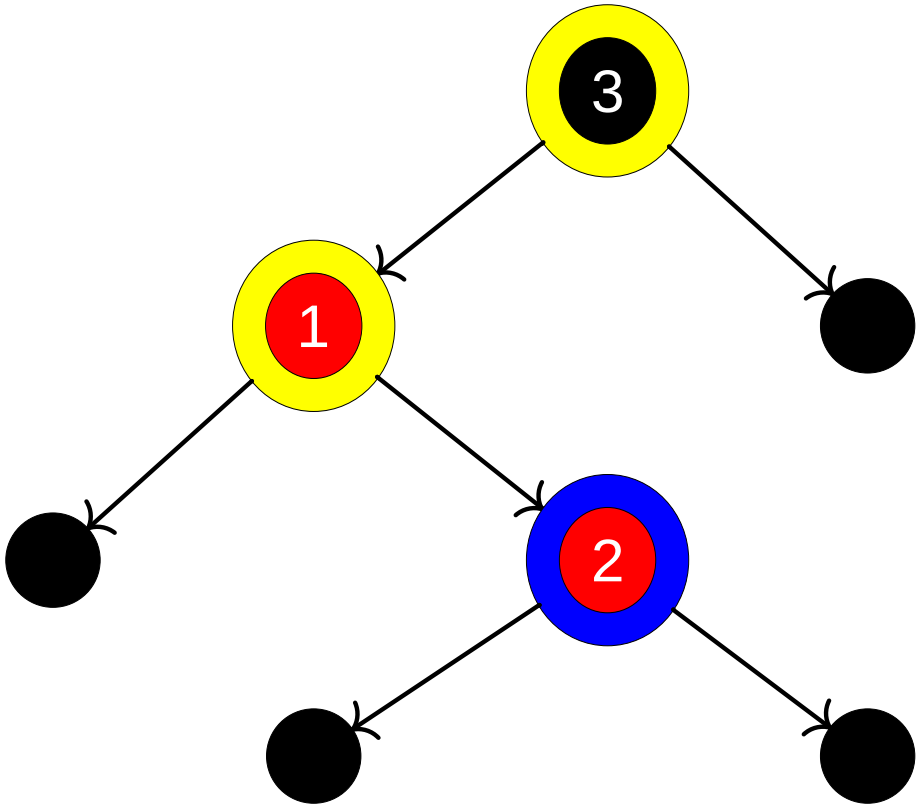

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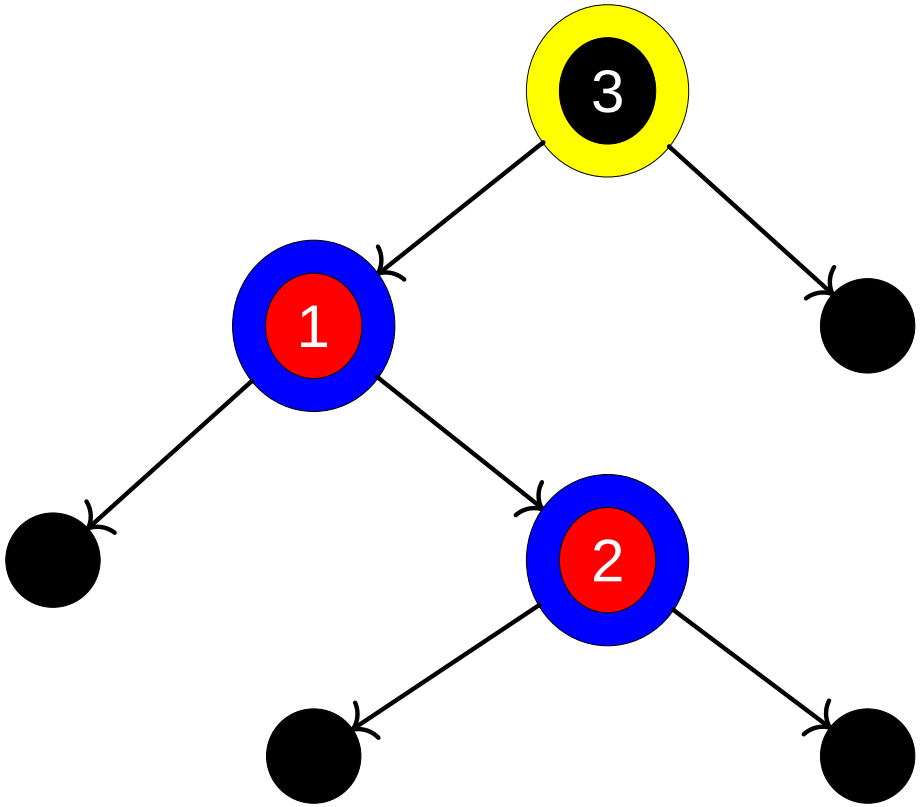

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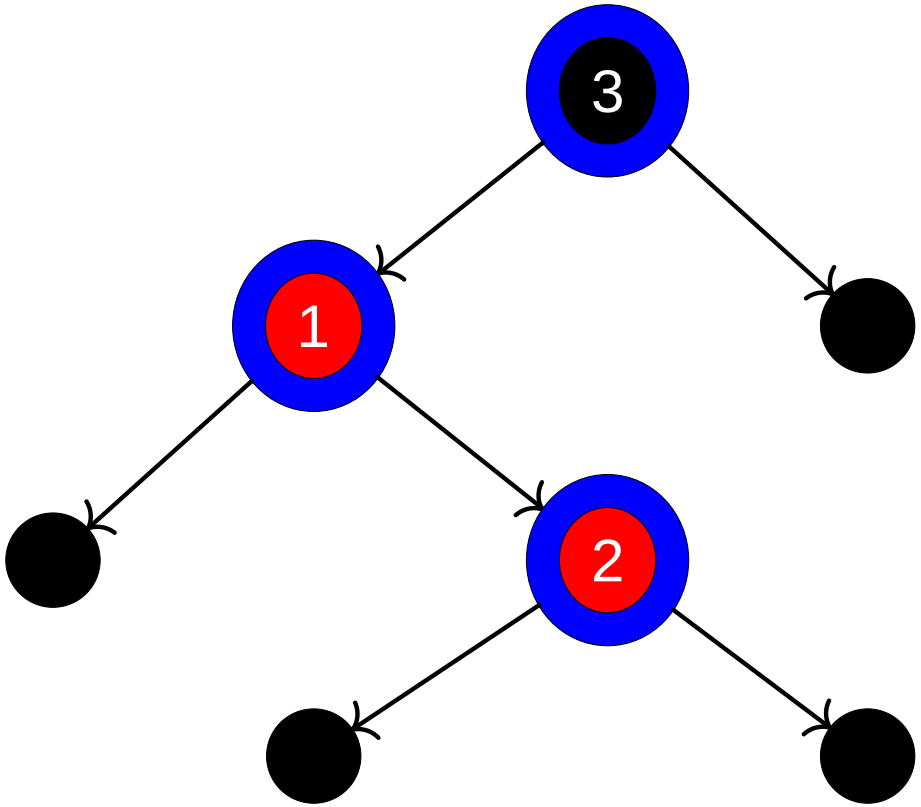

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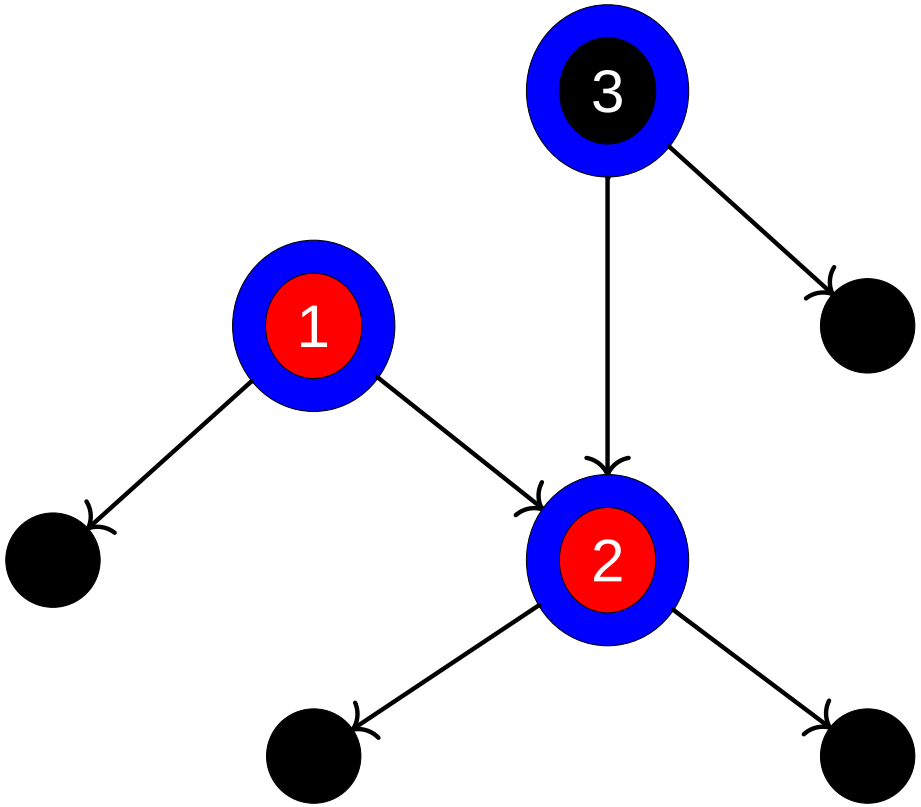

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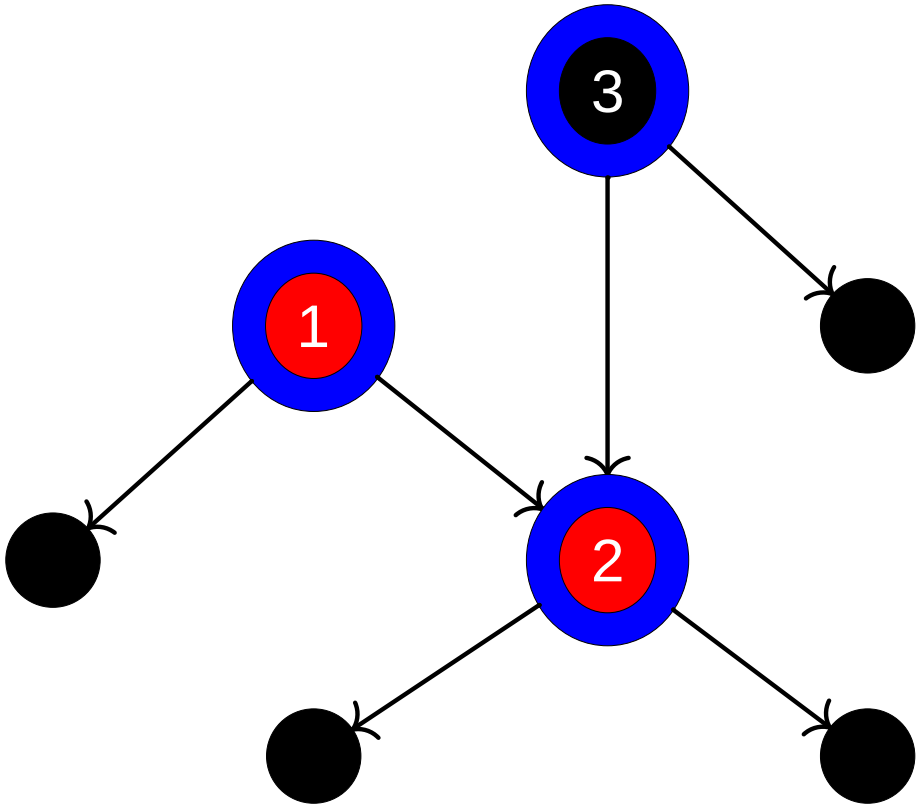

Looking Ahead

Plan

- implement all three algorithms
- compare their performance

Challenges

- adjust algorithm for AVL trees to red-black trees
- modify red-black tree algorithms of [Cormen, Leiserson, Rivest and Stein, 2001]
- when a process unlocks a node, which of the processes that are waiting to lock the node is chosen? (not addressed in the paper, PhD thesis is not available)