Name:	Student no.:

CSE 4214

Midterm Examination

Thursday, October 23, 2008

Instructions

- **This is a closed book exam.** You are permitted one 8 ½ by 11 inch sheet of handwritten personal notes, as well as a calculator.
- **Attempt all questions, and give your answer in the space provided.** There are four questions on this exam, worth a total of 40 marks.
- **Read each question carefully before answering.** If you find any question unclear, **state your assumptions with your answer**. You will not be penalized for any reasonable assumption.
- Time limit: **80 minutes.**

Question 1 (16 marks)

Hints for question 1:

$$\int_0^T \sin(\pi t/T)dt = 2T/\pi, \quad \int_0^T \sin^2(\pi t/T)dt = T/2.$$

Consider the following modulation signal:

$$s_0(t) = \begin{cases} \sin(\pi t/T), & 0 \le t \le T; \\ 0, & t < 0, t > T. \end{cases}$$

a. (8 marks) Suppose $s_1(t) = -s_0(t)$ for all t. Find the probability of error in terms of erfc, assuming that the signal is received in the presence of additive white Gaussian noise with power spectral density $N_0/2$, using the optimal decision rule, and assuming the matched filter is matched to $s_0(t)$.

b. (8 marks) For the same $s_0(t)$ and $s_1(t)$ as in part a, say the matched filter is replaced by a filter having impulse response

$$h(t) = \begin{cases} 1, & 0 \le t \le T; \\ 0, & t < 0, t > T. \end{cases}$$

Find the probability of error in terms of erfc using the optimal decision rule. Is this better or worse than the case in part a? Explain.

Question 2 (9 marks)

a. (4 marks) Suppose f(t) and g(t) each satisfy the Nyquist criterion for zero inter-symbol interference. Show that f(t)g(t) also satisfies the Nyquist criterion.

b. (1 mark) Data is transmitted at a rate of 10 kilobits per second. Assuming binary signaling, give the signaling time (T).

c. (2 marks) For the system in part b, what is the minimum bandwidth (W), counting only positive frequencies, so that the Nyquist criterion is satisfied?

d. (2 marks) For the system in part b, suppose 7.5 kHz of bandwith is available, counting only positive frequencies. Can a raised-cosine pulse be used in this amount of bandwidth? If so, give the largest possible value for the excess bandwidth. If not, explain why not.

Question 3 (10 marks)

a. (2 marks) What conditions must a random process satisfy in order to be a wide-sense stationary random process?

b. (4 marks) Let X(t) be a wide-sense stationary random process with mean $\mu = 0$ and autocorrelation $R_X(\tau)$. Let Y(t) = X(t) + t. Is Y(t) wide-sense stationary? Explain.

c. (4 marks) Let Z(t) be a random process where Z(t) = At, where A is the outcome of a single roll of a fair die (i.e., $f_A(a) = 1/6$ if a = 1, 2, …, 6). Find the probability density function $f_{Z(t)}(z)$.

Question 4 (5 marks)

Consider the signal sets:

#1	$s_0(t) = \begin{cases} -1, & 0 \le t < T/2 \\ 1, & T/2 \le t < T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = 0$
#2	$s_0(t) = \begin{cases} t, & 0 \le t \le T \\ 0, & t < 0, t > T \end{cases}$	$s_1(t) = -s_0(t)$

a. (2 marks) Sketch the impulse response of the filter matched to $s_0(t)$ for both signals.

b. (3 marks) Find the average energy per bit for both signals.

Hint: $\int t\,dt = \frac{1}{2}t^2$