

## Combinational Logic: Design of a 1-bit adder (2)

Step 2: Derive the Boolean expression for each output from the truth table

| INPUTS |  | OUTPUTS |  |  |
| :--- | :--- | :--- | :--- | :--- |
| a | b | c <br> (CarryIn) | CarryOut | Sum |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |



> Sum $=\bar{a} \bar{b} c+\bar{a} b \bar{c}+a \bar{b} \bar{c}+a b c$
> Carry-Out $=\bar{a} b c+a \bar{b} c+a b \bar{c}+a b c$

## Combinational Logic: Design of a 1-bit adder (3)

Step 3: Simplify the Boolean expression

$$
\text { Carry-Out }=\bar{a} b c+a \bar{b} c+a b \bar{c}+a b c=b c+a c+a b
$$

$$
\operatorname{Sum}=(\bar{a} \bar{b}+a b) c+(a \bar{b}+\bar{a} b) \bar{c}=\overline{(a \bar{b}+\bar{a} b)} c+(a \bar{b}+\bar{a} b) \bar{c}
$$

Step 4: Implement the simplified Boolean expression using OR, AND, and NOT gates


Activity: Implement the hardware for the Sum output of the 1-bit adder

## Agenda for Today

- Concerns from Prof. Roumani
- 1-bit ALU - Logic Design

Patterson: Appendix C

## Prof. Roumani's Concerns

- Not enough students are doing the pre-lab activities at home and as a consequence not very many students are completing the Lab exercises on time
- If you labs have been manually marked, you can pick them up from the TA either at the next lab session or during his office hours (W 16:00 - 17:00) - NO ONE has done this
- All labs have been posted on ePost for Labs A and B. Manually marked Lab C's are to be posted in a few days


## 1-bit adder

- Recall the digital circuit of a 1-bit adder
- We will enhance the 1-bit adder to develop a prototype ALU for MIPS


Digital Circuit of a 1-bit adder


Schematic of a 1-bit adder

## 1-bit ALU with AND, OR, and Addition

- The 1-bit adder is supplemented with AND and OR gates
- A multiplexer controls which gate is connected to the output


1-bit ALU with AND, OR, and Addition capability

| ALU Control Lines |  | Result |
| :--- | :--- | :--- |
| Carry In | Operation |  |
| $\mathbf{0}$ | $\mathbf{0}=(\mathbf{0 0})_{\mathbf{t w o}}$ | add |
| $\mathbf{0}$ | $\mathbf{1}=(\mathbf{0 1})_{\mathbf{t w o}}$ | OR |
| $\mathbf{0}$ | $\mathbf{2}=\mathbf{( 1 0})_{\mathbf{t w o}}$ | AND |



Schematic

## 32-bit ALU w/ AND, OR, and ADD

- The 1-bit ALU can be cascaded together to form a 32 bit ALU
- Which operation is performed is controlled by the Operation bus

| ALU Control Lines |  | Result |
| :--- | :--- | :--- |
| Carry In | Operation |  |
| $\mathbf{0}$ | $\mathbf{0}=\mathbf{( 0 0})_{\mathrm{two}}$ | add |
| $\mathbf{0}$ | $\mathbf{1}=(\mathbf{0 1})_{\mathrm{two}}$ | OR |
| $\mathbf{0}$ | $\mathbf{2}=\mathbf{( 1 0})_{\mathrm{two}}$ | AND |

- The designed 32-bit ALU is still missing the subtraction, slt (set if less than), and conditional branch operations



## 1-bit ALU with AND, OR, Addition, and Subtraction

- Recall that subtraction is performed using 2's complement arithmetic
- We calculate the 2's compliment of the sub-operand and add to the first operand


| ALU Control Lines |  |  | Result |
| :--- | :--- | :--- | :--- |
| $\mathbf{B i n v e r t}$ | Carry In | Operation |  |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{2}=\mathbf{( 1 0})_{\mathrm{two}}$ | AND |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}=\mathbf{( 0 1})_{\mathrm{two}}$ | $\mathbf{O R}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}=\mathbf{( 0 0})_{\mathrm{two}}$ | $\mathbf{a d d}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}=\mathbf{( 0 0})_{\mathrm{two}}$ | $\mathbf{s u b}$ |

1-bit ALU with AND, OR, Addition, and Subtraction capability

## 1-bit ALU with AND, OR, Add, Sub, and SLT (1)

- Since we need to perform one more operation, we increase the number of inputs at the multiplexer by 1 and label the new input as Less
- SLT operation: if (a<b), set Less to 1 $=>$ if $(a-b)<0$, set Less to 1
- SLT operation can therefore be expressed in terms of a subtraction between the two operands.
- If the result of subtraction is negative, set Less to 1 .
- How do we determine if the result is negative?


1-bit ALU with AND, OR, Add, Sub, and SLT capability

- The 1-bit ALU's can be cascaded together to form a 32 bit ALU
- Operations are controlled by the Operation bus

| ALU Control Lines |  |  |  |
| :--- | :--- | :--- | :--- |
| Binvert | Carry In | Operation |  |
| 0 | 0 | $0=(00)_{\text {two }}$ | Add sum(a,b) |
| 0 | 0 | $1=(01)_{\text {two }}$ | OR (a+b) |
| 0 | 0 | $2=(10)_{\text {two }}$ | AND (a•b) |
| 1 | 1 | $0=(00)_{\text {two }}$ | Subtract $(\mathrm{a}-\mathrm{b})$ |
| 1 | 1 | $\mathbf{3}=(11)_{\text {two }}$ | SLT <br> Set Result0 if $(\mathrm{a}<\mathrm{b})$ |

- Note that Binvert is always the same as Carry. In
- To test equality between $a$ and $b$, subtract $b$
 from a and check if the result is 0 . W5-W

32-bit ALU w/ And, OR, Add, Subtract, SLT, and Equality Test

| ALU Control Lines |  | Result |  |
| :--- | :--- | :--- | :--- |
| Binvert | Carry In | Operation |  |
| 0 | 0 | $0=(00)_{\mathrm{two}}$ | Add sum $(\mathrm{a}, \mathrm{b})$ |
| 0 | 0 | $1=(01)_{\mathrm{two}}$ | OR (a+b) |
| 0 | 0 | $2=(10)_{\mathrm{two}}$ | AND (a•b) |
| 1 | 1 | $0=(00)_{\mathrm{two}}$ | Subtract $(\mathrm{a}-\mathrm{b})$ |
| 1 | 1 | $3=(11)_{\mathrm{two}}$ | SLT <br> if $(\mathrm{a}<\mathrm{b})$ <br> Result0 $=1$ |
| 1 | 1 | $0=(00)_{\mathrm{two}}$ | Test Equality <br> Zero $=1 \mathrm{if}(\mathrm{a}=\mathrm{b})$ |



32-bit ALU w/ And, OR, Add, Subtract, SLT, and Equality Test

U_ALU1bitInvSltTz
A $\bar{L} U 1$ bitInvSltTz.SchDoc



W5-W

