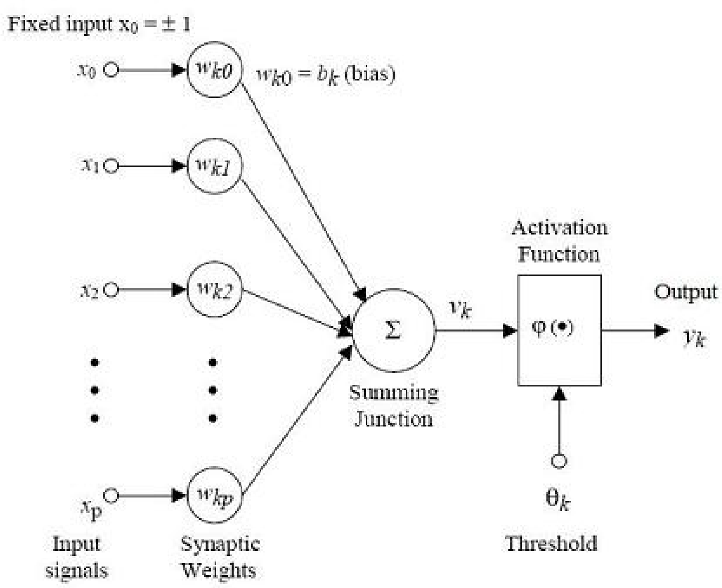
Neural Networks - The Mathematical Model

Once modeling an artificial functional model from the biological neuron, we must take into account three basic components. First off, the synapses of the biological neuron are modeled as weights. Let's remember that the synapse of the biological neuron is the one which interconnects the neural network and gives the strength of the connection. For an artificial neuron, the weight is a number, and represents the synapse. A negative weight reflects an inhibitory connection, while positive values designate excitatory connections. The following components of the model represent the actual activity of the neuron cell. All inputs are summed altogether and modified by the weights. This activity is referred as a linear combination. Finally, an activation function controls the amplitude of the output. For example, an acceptable range of output is usually between 0 and 1, or it could be -1 and 1. Mathematically, this process is described in the figure



From this model the interval activity of the neuron can be shown to be:

$$v_k = \sum_{j=1}^{p} w_{kj} x_j$$

The output of the neuron, yk, would therefore be the outcome of some activation function on the value of vk.

Activation functions

As mentioned previously, the activation function acts as a squashing function, such that the output of a neuron in a neural network is between certain values (usually 0 and 1, or -1 and 1). In general, there are three types of activation functions, denoted by Φ(.) . First, there is the Threshold Function which takes on a value of 0 if the summed input is less than certain threshold value (v), and the value 1 if the summed input is greater than or equal to the threshold value.

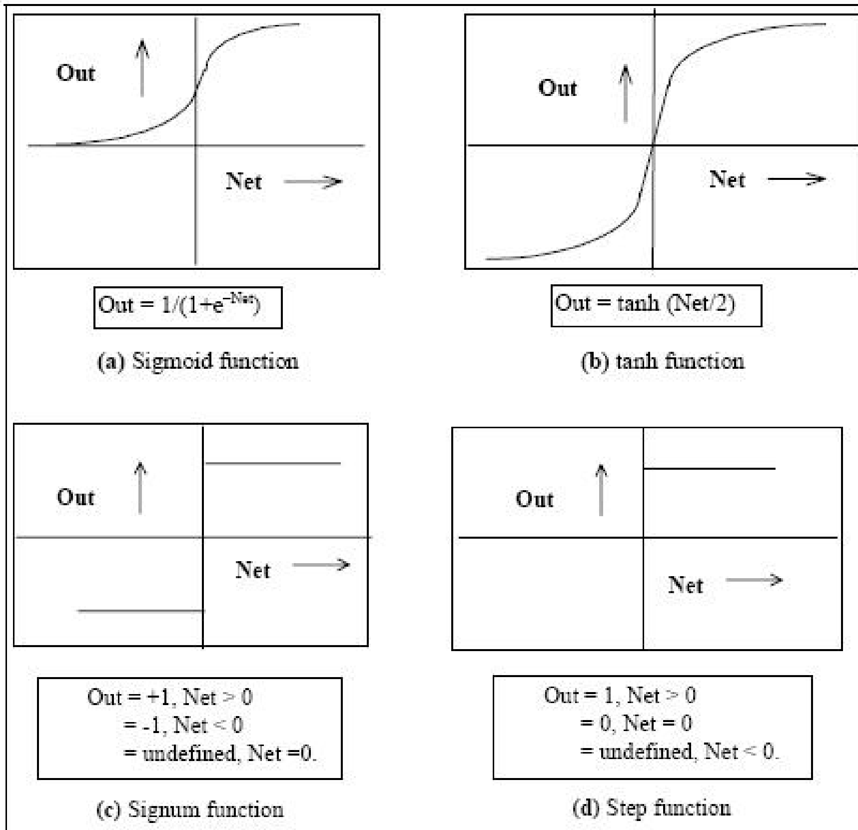
$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases}$$

Secondly, there is the Piecewise-Linear function. This function again can take on the values of 0 or 1, but can also take on values between that depending on the amplification factor in a certain region of linear operation.

Thirdly, there is the sigmoid function. This function can range between 0 and 1, but it is also sometimes useful to use the -1 to 1 range. An example of the sigmoid function is the hyperbolic tangent function.

$$\varphi(v) = \begin{cases} 1 & v \geq \frac{1}{2} \\ v & -\frac{1}{2} > v > \frac{1}{2} \\ 0 & v \leq -\frac{1}{2} \end{cases}$$

$$\varphi(v) = \tanh\left(\frac{v}{2}\right) = \frac{1-\exp(-v)}{1+\exp(-v)}$$



Common non-linear functions used for synaptic inhibition. Soft non-linearity: (a) Sigmoid and (b) tanh; Hard non-linearity: (c) Signum and (d) Step.

The artificial neural networks which we describe are all variations on the parallel distributed processing idea. The architecture of each neural network is based on very similar building blocks which perform the processing. We next discuss these processing units and different neural network topologies.