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Rough set approach to knowledge-based decision support

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Abstract

Rough set theory is a new approach to decision making in the presence of uncertainty and vagueness. Basic concepts of rough set theory will be outlined and its possible application will be briefly discussed. Further research problems will conclude the paper. © 1997 Elsevier Science B.V.

Keywords: Rough sets; Fuzzy sets; Decision support

1. Introduction

The rough set concept proposed by the author in [51] is a new mathematical approach to imprecision, vagueness and uncertainty. The rough set philosophy is founded on the assumption that with every object of the universe of discourse we associate some information (data, knowledge); e.g., if objects are patients suffering from a certain disease, symptoms of the disease form information about patients. Objects characterized by the same information are indiscernible (similar) in view of the available information about them. The indiscernibility relation generated in this way is the mathematical basis of rough set theory.

Any set of all indiscernible (similar) objects is called elementary set, and form basic granule (atom) of knowledge about the universe. Any union of some elementary sets is referred to as crisp (precise) set – otherwise a set is rough (imprecise, vague).

Consequently each rough set has boundary-line cases, i.e., objects which cannot be with certainty classified as members of the set or of its complement. Obviously crisp sets have no boundary-line elements at all. That means that boundary-line cases cannot be properly classified by employing the available knowledge.

Thus, the assumption that objects can be 'seen' only through the information available about them leads to the view that knowledge has a granular structure. Due to the granularity of knowledge some objects of interest cannot be discerned and appear as the same (or similar). As a consequence vague concepts, in contrast to precise concepts, cannot be characterized in terms of information about their elements. Therefore, in the proposed approach we assume that any vague concept is replaced by a pair of precise concepts – called the lower and the upper approximation of the vague concept. The lower approximation consists of all objects which surely belong to the concept and the upper approximation contains all objects which possible belong to the concept. Obviously, the difference between the upper

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and the lower approximation constitute the boundary region of the vague concept. Approximations are two basic operations in the rough set theory.

The basic operations of rough set theory are used to discover fundamental patters in data. Thus, in a certain sense the rough set methodology refers to machine learning, knowledge discovery, statistics and inductive inference. However, interpretation of the obtained results lies outside the theory and can be used in many ways.

Rough set theory overlaps to a certain degree many other mathematical theories. Particularly interesting is the relationship with fuzzy set theory and Dempster–Shafer theory of evidence. The concepts of rough set and fuzzy set are different since they refer to various aspects of imprecision [53], whereas the connection with theory of evidence is more substantial [63]. Besides, rough set theory is related to discriminant analysis [33], Boolean reasoning methods [64] and others. More details concerning these relationships can be found in the references. Despite of these connections rough set theory is an independent and mature discipline, in its own rights.

For basic ideas of the rough set theory the reader is referred to [52]. Many interesting applications of this approach are presented and discussed in [40,41,66,93]. The relationship between the rough set theory and decision analysis is presented in [54,67]. The present state of the rough set theory and its further perspectives are discussed in [54]. An extensive study of various mathematical models of uncertainty can be found in [14].

2. Basic concepts of the rough set theory

2.1. Indiscernibility relation

As mentioned in the introduction, the starting point of the rough set theory is the indiscernibility relation, generated by information about objects of interest. The indiscernibility relation is intended to express the fact that due to the lack of knowledge we are unable to discern some objects employing the available information. That means that, in general, we are unable to deal with single objects but we have to consider clusters of indiscernible objects, as fundamental concepts of our theory.

The indiscernibility relation may be formulated in quite general mathematical framework, but for the sake of intuition we will define it referring to an *information table* called also an *information system* or an *attribute-value table.*

Table 1

Store	E	Q	L	P
1	high	good	no	profit
2	med.	good	no	loss
3	med.	good	no	profit
4	no	avg.	no	loss
5	med.	avg.	yes	loss
6	high	avg.	yes	profit

An example of a simple information table is presented in Table 1. In Table 1 six stores are characterized by four attributes:

E empowerment of sales personnel,
Q perceived quality of merchandise,
L high traffic location,
P store profit or loss.

Let us observe that each store has a different description in terms of attributes E, Q, L and P, thus all stores may be distinguished (discerned) employing information provided by all attributes. However, stores 2 and 3 are indiscernible in terms of attributes E, Q and L, since they have the same values of these attributes. Similarly, stores 1, 2 and 3 are indiscernible with respect to attributes Q and L, etc.

Each subset of attributes determines a partition (classification) of all objects into classes having the same description in terms of these attributes. For example, attributes Q and L aggregate all stores into the following classes {1,2,3}, {4}, {5,6}. Thus, each information table determines a family of classification patterns which are used as a basis of further considerations.

The above consideration can be presented in a more formal way as follows. Let U be a finite set of *objects* – called the *universe* – and let A be a finite set of *attributes.* With every attribute $a \in A$ set of its *values* V_a is associated. Each attribute a determines a function $f_a : U \to V_a$. With every subset of attributes B of A we associate an indiscernibility relation on U, denoted $I(B)$ and defined thus:

$$I(B) = \{(x,y) \in U \times U : f_a(x) = f_a(y), \forall a \in B\}.$$

It is easily seen that the indiscernibility relation defined in this way is an equivalence relation. The family of all equivalence classes of the relation $I(B)$ will be denoted by $U/I(B)$, in short U/B, and an equivalence class containing an element x will be denoted as $I(B)(x)$, in short $B(x)$. If (x, y) belongs to $I(B)$ we will say that x and y are *B-indiscernible*. Equivalence classes of the relation $I(B)$ (or blocks of the partition U/B) are refereed to as *B-elementary sets*. In the rough set approach the elementary sets are the basic building blocks (concepts) of our knowledge about reality.

The indiscernibility relation will be used next to define basic concepts of rough set theory.

2.2. Approximations

Suppose we are interested in the following problem: what are the characteristic features of stores having profit (or loss) in view of information available in Table 1. In other words, the question is whether we are able to describe set (concept) {1,3,6} (or {2,4,5}) in terms of attributes E, Q and L. It can be easily seen that this question cannot be answered uniquely in our case, since stores 2 and 3 display the same features in terms of attributes E, Q and L, but store 2 makes a profit, whereas store 3 has a loss. Thus, information given in Table 1 is not sufficient to answer this question. However, we can give a partial answer to this question. Let us observe that if the attribute E has the value *high* for a certain store, then the store makes a profit, whereas if the value of the attribute E is *low*, then the store has a loss. Thus, in view of information contained in Table 1, we can say for sure that stores 1 and 6 make a profit, stores 4 and 5 have losses, whereas stores 2 and 3 cannot be classified as making a profit or having losses. Therefore we can give approximate answers only. Employing attributes E, Q and L, we can say that stores 1 and 6 *surely* make a profit, i.e., *surely* belong to the set {1,3,6}, whereas stores 1, 2, 3 and 6 *possibly* make a profit, i.e., possibly belong to the set {1,3,6}. We will say that the set {1,6} is the *lower approximation* of the set (concept) {1,3,6}, and the set {1,2,3,6} is the *upper approximation* of the set {1,3,6}. The set {2,3}, being the difference between the upper approximation and the lower approximation is referred to as the *boundary region* of the set {1,3,6}.

The above ideas can be presented more precisely in the following manner. Let U be the universe, X a subset (a concept) of the universe, and let B be a subset of A. Let us define now the following operations on sets:

$$B_*(X) = \{x \in U : B(x) \subseteq X\},$$

$$B^*(X) = \{x \in U : B(x) \cap X \neq \varnothing\},$$

assigning to every subset X of the universe U two sets $B_*(X)$ and $B^*(X)$ called the *B-lower* and the *B-upper approximation* of X, respectively. The set

$$\mathrm{BN}_B(X) = B^*(X) - B_*(X)$$

will be referred to as the *B-boundary region* of X.

If the boundary region of X is the empty set, i.e., $\mathrm{BN}_B(X) = \varnothing$, then the set X will be called *crisp* (*exact*) with respect to B; in the opposite case, i.e., if $\mathrm{BN}_B(X) \neq \varnothing$, the set X will be referred to as *rough* (*inexact*) with respect to B.

Rough set can be also characterized numerically by the following coefficient

$$\alpha_B(X) = \frac{|B_*(X)|}{|B^*(X)|},$$

called the *accuracy of approximation*, where $|X|$ denotes the cardinality of X, $X \neq \varnothing$. Obviously, $0 \leq \alpha_B(X) \leq 1$. If $\alpha_B(X) = 1$, X is *crisp* with respect to B (X is *precise* with respect to B), and otherwise, if $\alpha_B(X) < 1$, X is *rough* with respect to B (B is *vague* with respect to X). For example, the accuracy of approximation of the set {1,3,6} is $2/4 = 1/2$.

2.3. Rough membership

A vague concept has boundary-line cases, i.e., elements of the universe which cannot be – with *certainty* – classified as elements of the concept. Hence, *uncertainty* is related to the *membership* of elements to a set. Therefore, in order to discuss the problem of uncertainty from the rough set perspective we have to define the membership function related to the rough set concept (the *rough membership function*) [53].

The rough membership function can be defined employing the indiscernibility relation as

$$\mu_X^B(x) = \frac{|X \cap B(x)|}{|B(x)|}.$$

Obviously

$$\mu_X^B(x) \in [0,1].$$

The value of the membership function $\mu_X(x)$ may be interpreted in some cases as a conditional probability, and can be understood as a degree of *certainty* to which x belongs to X (or $1 - \mu_X(x)$, as a degree of uncertainty).

Let us notice that the value of the membership function is computed from available data, and not assumed, as in the case of the fuzzy membership function.

The rough membership function, can be used to define approximations and the boundary region of a set, as shown below:

$$B_*(X) = \{x \in U : \mu_X^B(x) = 1\},$$

$$B^*(X) = \{x \in U : \mu_X^B(x) > 0\},$$

$$\mathrm{BN}_B(X) = \{x \in U : 0 < \mu_X^B(x) < 1\}.$$

One can see from the above definitions that there exists a strict connection between vagueness and uncertainty in the rough set theory. As we mentioned above vagueness is related to sets, while uncertainty is related to elements of sets.

Thus approximations are necessary when speaking about vague concepts, whereas rough membership is needed when uncertain data are considered.

2.4. Dependency of attributes

Another important issue in data analysis is discovering dependencies between attributes. Intuitively, a set of attributes B depends totally on a set of attributes C, denoted $C \Rightarrow B$, if all values of attributes from B are uniquely determined by values of attributes from C. In other words, B depends totally on C, if there exists a functional dependency between values of B and C. In Table 1 there are no total dependencies whatsoever.

Formally, dependency can be defined in the following way. Let B and C be subsets of A. We say that B *depends totally* on C, if and only if $I(C) \subseteq I(B)$. That means that the partition generated by C is finer than the partition generated by B. Notice, that the concept of dependency discussed above corresponds to that considered in relational databases.

We would need also a more general concept of dependency of attributes, called a *partial dependency* of attributes.

Formally, the above idea can be formulated as follows. Let B and C be subsets of A. We say that B *depends in degree* k, $0 \le k \le 1$, on C, denoted $C \Rightarrow_k B$, if

$$k = \frac{|\mathrm{POS}_C(B)|}{|U|},$$

where

$$\mathrm{POS}_C(B) = \bigcup_{X \in U/B} C_*(X).$$

The expression $\mathrm{POS}_C(B)$, called a *positive region* of the partition U/B with respect to C, is a set of all elements of U that can be uniquely classified to blocks of the partition U/B, by means of C.

In other words B is *totally* (*partially*) dependent on C, if *all* (*some*) elements of the universe U can be uniquely classified to blocks of the partition U/B, employing C.

2.5. Reduction of attributes

We often face a question whether we can remove some data from an information table preserving its basic properties, that is, whether a table contains some superfluous data.

In order to express the above idea more precisely we need some auxiliary notions. Let B be a subset of A and let a belong to B.

1. We say that a is *superfluous* in B if $I(B) = I(B - \{a\})$; otherwise a is *indispensable* in B.
2. Set B is *independent* (*orthogonal*) if all its attributes are indispensable.
3. Subset B' of B is a *reduct* of B if B' is independent and $I(B') = I(B)$.

Thus, a reduct is a set of attributes that preserves partition. It means that a reduct is a minimal subset of attributes that enables the same classification of elements of the universe as the whole set of attributes. In other words, attributes that do not belong to a reduct are superfluous with regard to classification of elements of the universe.

Reducts have several important properties. A very important one is the following.

First, we define a notion of a core of attributes. Let B be a subset of A. The core of B is a set off all indispensable attributes of B. The following is an important property, connecting the notion of the core and reducts

$$\text{Core}(B) = \bigcap \text{Red}(B),$$

where $\text{Red}(B)$ is the set off all reducts of B.

Because the core is the intersection of all reducts, it is included in every reduct, i.e., each element of the core belongs to some reduct. Thus, in a sense, the core is the most important subset of attributes, for none of its elements can be removed without affecting the classification power of attributes.

Complexity of computing all reducts in an information system is rather high. However, in many applications we do not need to compute all reducts, but only some of them, satisfying specific requirements, which is much simpler. There are many approaches to compute reducts. For details see for example [2,38,65,87].

3. Decision tables and decision algorithms

If in an information table we distinguish two classes of attributes, called *condition* and *decision* attributes, then such a table is called a *decision* table. In Table 1 the attribute P can be regarded as a decision attribute, whereas attributes E, Q and L are condition attributes. Condition attributes specify decisions which should be performed if conditions, determined by condition attributes, are satisfied. In fact, when reducing condition attributes we usually would like to preserve the dependency between condition and decision attributes, which enable us to make decision using less information,

The concept of the reduct can be easily generalized in such a way that it preserves not necessarily partitions generated by attributes, but other features, for example the degree of dependency between condition and decision attributes. We will skip a detailed consideration of this problem here, and illustrate it only by means of the following example.

It can be easily seen that there are two reducts $\{E,Q\}$ and $\{E,L\}$ of the set of attributes $\{E,Q,L\}$ which preserve the degree of dependency between the condition and the decision attributes, i.e., $\{E,Q\} \Rightarrow_{2/3}\{P\}$ and $\{E,L\} \Rightarrow_{2/3}\{P\}$.

Table 2

Store	E	Q	P
1	high	good	profit
2	med.	good	loss
3	med.	good	profit
4	low	avg.	loss
5	med.	avg.	loss
6	high	avg.	profit

Sometimes the degree of dependency between condition and decision attributes is called a *quality of approximation* of classification induced by the set of decision attributes – by classification generated by the condition attributes.

Obviously the core of the set of attributes $\{E,Q,L\}$ is the attribute E, which is, in a certain sense, the most important attribute describing stores, while the attributes L and Q can be mutually exchanged. Consequently, instead of Table 1, we can use either Table 2 or Table 3.

Decision tables can be also understood as a set of *decision rules*. For example Table 2 can be represented by the following set of decision rules:

If $(E$, high) and $(Q$, good), then $(P$, profit);
If $(E$, med.) and $(Q$, good), then $(P$, loss);
If $(E$, med.) and $(Q$, good), then $(P$, profit);
If $(E$, low) and $(Q$, avg.), then $(P$, loss);
If $(E$, med.) and $(Q$, avg.), then $(P$, loss);
If $(E$, high) and $(Q$, avg.), then $(P$, profit).

Using a rough set technique the above decision rules can be simplified further, leading to a minimal set of decision rules shown below:

If $(E$, high), then $(P$, profit);
If $(E$, low), then $(P$, loss);
If $(E$, med.) and $(Q$, avg.), then $(P$, loss);

or in a more concise form

If $(E$, high), then $(P$, profit);
If $(E$, low) or ((E, med.) and $(Q$, avg.)), then $(P$, loss).

However, there are essential methodological and practical differences between decision tables and sets

Table 3

Store	E	L	P
1	high	no	profit
2	med.	no	loss
3	med.	no high	profit
4	low	no	loss
5	med.	yes	loss
6	high	yes	profit

decision rules (decision algorithms). In the of case of decision tables we think about a collection of data, which can be treated by means of various algebraic or statistical methods. Whereas decision rules are logical expressions (implications), of the form 'if ... then', which belong to a entirely different realm, and require, in contrast to decision tables, logical means to deal with. Consequently algorithms based on decision tables and on decision rules are totally distinct. We will refrain to discuss this problem in details here. The interested reader is advised to consult [52].

4. Rough sets and decision analysis

Any decision problem involves a set of objects, e.g., actions, states, processes, competitors, etc. In general, objects can be anything we can think of. The objects are described by attribute-value pairs. As we mentioned already, such sets of data can be represented by a table, rows of which correspond to objects, columns to attributes and entries of the table are attribute values.

The table represents some facts about the decision problem. In particular, it may represent opinions of agents, groups of agents, decision makers, etc.

The aim of the decision analysis is to answer the following two basic questions. The first question is to *explain* decision in terms of circumstances in which the decision has been made. The second, is to give a *prescription* how to make a decision under specific circumstances. Prescription is mainly based on decision rules derived from a decision table. In this sense, the rough set approach is similar to the inductive learning approach, however, the former one is going far beyond the latter because in the rough set approach, the prescription task is preceded by the explanation which gives pertinent information useful for decision support. Besides, optimization of decision rules is also of great importance, but we will not consider this issue here.

Rough set theory offers mathematical tools to answer the above mentioned questions, and seems particularly suited to analyze this kind of problems. In other words rough set theory offers techniques to generate minimal sets of decision rules from specification of the decision process.

Let us mention however, that there are some differences between the rough set approach and the 'classical' decision analysis to decision problems. In decision analysis we distinguish three basis classes of decision problems:

1. multi-attribute sorting problem,
2. multi-attribute, multi-sorting problem,
3. multi-attribute description of objects.

In our terminology the difference between multi-attribute sorting problem and multi-attribute, multi-sorting problem is that in the first case we have only one decision attribute, whereas in the second case many decision attributes are allowed. Thus, in the rough set approach we do not need to distinguish these classes formally, because the first class is a special case of the second one, and there is no necessity to consider them separately. Of course, there are many possible interpretations of a decision problem considered. For example, decision attributes may represent agents involved in a decision process, prescriptions, decisions, opinions or explanation. In rough set theory the interpretation does not influence the formal model and all sorting problems may be treated in a unified manner, regardless of the interpretation.

Similarly, in the case of multi-attribute description of objects, many interpretations are possible. For example, agents may be represented by objects or attributes, or may not be represented explicitly in the formal model at all. The model can represent individual or group decisions etc.

Summing up, rough set theory offers a unified formal approach to all above mentioned classes of decision problems, despite of interpretation. More about the application of the rough set theory in decision analysis can be found in [54,67].

5. Some remarks on applications

Rough set theory has found many interesting applications, The rough set approach seems to be of fundamental importance to AI and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, expert systems, inductive reasoning and pattern recognition. It seems of particular importance to decision support systems.

The main advantage of rough set theory is that it does not need any preliminary or additional information about data – like probability in statistics, or basic probability assignment in Dempster–Shafer theory and grade of membership or the value of possibility in fuzzy set theory.

The rough set theory has been successfully applied in many real-life problems in medicine, pharmacology, engineering, banking, financial and market analysis and others. Some exemplary applications are listed below.

There are many applications in medicine [18,72–75,80]. In pharmacology the analysis of relationships between the chemical structure and the antimicrobial activity of drugs [34–37] has been successfully investigated. Banking applications include evaluation of a bankruptcy risk [70,71] and market research [12,95]. Very interesting results have been also obtained in speaker independent speech recognition [7–11] and acoustics [29–32]. The rough set approach seems also important for various engineering applications, like diagnosis of machines using vibroacoustics, symptoms (noise, vibrations) [48–50], material sciences [24] and process control [42,45,47,58,77,91,95]. Application in linguistics [28,46] and environment [46], databases [5,6,23,60,90] are other important domains.

More about applications of the rough set theory can be found in [40,66,86,93]. Besides, many other fields of application, e.g., time series analysis, image processing and character recognition, are being extensively explored.

Application of rough sets requires suitable software. Many software systems for workstations and personal computers based on rough set theory have been developed. The most known include LERS [15], Rough DAS and Rough Class [13] and DATALOGIC [78]. Some of them are available commercially.

One of the most important and difficult problem in software implementation of the presented approach is optimal decision rule generation from data. Many various approaches to solve this task can be found in [1,3,17,61,64,85,87]. The relation to other methods of rule generation is dwelt in [17].

6. Conclusion

The rough set theory turned out to be a very useful tool for decision support systems, especially when vague concepts and uncertain data are involved in the decision process.

The theory has many important advantages. Some of them are listed below:

- provides efficient algorithms for finding hidden patterns in data;
- finds minimal sets of data (data reduction);
- evaluates significance of data;
- generates minimal sets of decision rules from data;
- it is easy to understand;
- offers straightforward interpretation of obtained results;
- most algorithms based on the rough set theory are particularly suited for parallel processing, but in order to exploit this feature fully, a new hardware is necessary.

Although rough set theory has many achievements to its credit, nevertheless several theoretical and practical problems require further attention.

Especially important is widely accessible efficient software development for rough set based data analysis, particularly for large collections of data analysis.

Despite of many valuable methods of efficient, optimal decision rule generation methods from data, developed in recent years based on rough set theory, more research here is needed, particularly, when quantitative attributes are involved. In this context also further discretization methods for quantitative attribute values are badly needed. Comparison to other similar methods still requires due attention, although important results have been obtained in this area. Particularly interesting seems to be a study of the relationship between neural network and rough set approach to feature extraction from data.

Last but not least, rough set computer is badly needed for more serious computations in decision support. Some research in this area is already in progress.

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