## Introduction

## to FSA and Regular Expressions

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## Introduction

- Regular Languages and Finite Automata are among the oldest topics in formal language theory (early ${ }^{\text {' 40) }}$
- Formal language theory uses algebra and set theory to define formal languages as a sequence of symbols
- RL and FA have a wide range of applications:
- Lexical analysis in programming language compilation
- Circuit design, text editing, pattern matching, ...
- More recently: parallel processing, image generation and compression, type theory for 00 languages, DNA computing, ...


## Naïve definitions

- Basically, a regular expression is a pattern describing a certain amount of text
- A regular expression is a string that is used to describe or match a set of strings, according to certain syntax rules
- A regular expression, often called a pattern, is an expression that describes a set of strings. They are usually used to give a concise description of a set, without having to list all elements
- For example, the three strings Handel - Händel Haendel could be described by the pattern H(a|ä|ae)ndel


## Representations for languages

- A formal language is a language that is defined by precise mathematical or machine processable formulas.
- Formal languages generally have two aspects:
- the syntax of a language is what the language looks like (i.e. the set of possible expressions that are valid utterances in the language)
- the semantics of a language are what the utterances of the language mean (which is formalized in various ways, depending on the type of language in question)


## Representations for languages

- The branch of mathematics and computer science which studies exclusively the theory of language syntax is known as formal language theory
- In formal language theory, a language is nothing more than its syntax
- Questions of semantics are not addressed


## Formal languages and computability

- Strong connection with the computability theory, i.e. the branch of the theory of computation that studies which problems are computationally solvable using different models of computation
- The study of abstract machines and problems they are able to solve
- Typical questions asked about such formalisms include:
- What is their expressive power? (Can formalism X describe every language that formalism $Y$ can describe? Can it describe other languages?)
- What is their recognizability? (How difficult is it to decide whether a given word belongs to a language described by formalism X?)
- What is their comparability? (How difficult is it to decide whether two languages, one described in formalism $X$ and one in formalism Y , or in X again, are actually the same language?).


## Representations for languages

- We will discuss the two principal methods for defining languages: the generator and the recognizer
- In particular we will focus on a particular class of generators (grammars) and of recognizers (automata)
- There are many types of formal languages, some of them are very "simple", others are more "complex"
- It is possible to put them in a hierarchy
- Regular languages are the simplest formal languages:
- Their generators are the regular expressions
- Their recognizers are the finite state automata


## Automata theory: formal languages and formal grammars



Each category of languages or grammars is a proper subset of the category directly above it.

## Strings and Languages

- An alphabet is defined as any set of symbols
- Two examples:
the set of 26 upper and 26 lower case Roman letters (the Roman alphabet)
- the set $\{0,1\}$-> the binary alphabet
- Strings over an alphabet $\Sigma$ are defined as
- $\varepsilon$ (i.e. the empty string) is a string of $\Sigma$
- if $x$ is a string of $\Sigma$ and $a$ is in $\Sigma$, then $x a$ is in $\Sigma$ (concatenation)
- A language over $\Sigma$ is a set of string over $\Sigma$


## Operations on strings and languages

- Concatenations (or product):
if $x$ and $y$ are strings over an alphabet $\Sigma$, then $x y$ is
called the concatenation of $x$
Ex: if $x=a b$ and $y=c d$ then $x y=a b c d$
- Reversal:
$x^{R}$ is the string $x$ written in the reverse order
Ex: $x=a b c d$ then $x^{R}=d c b a$
- Closure:
$a^{0}=\varepsilon$
$a^{n}=a^{n-1}$ for $n \geq 1$
$a^{*}=U_{n \geq 0} a^{n}$
- Positive Closure:
$a^{+}=a a^{*}=\cup_{n \geq 1} a^{n}$


## Motivations

- How to represent a language $L$ ?
(e.g. when $L$ is infinite, that is contains an arbitrary number of strings)
- Two principal methods:
- Use a generative system, called grammar -> a set of rules that tell us which are the well-formed sentences in the language
- Use a device (an automaton) that for a given input string will halt and answer "yes" if the string belongs to the language


## Regular Sets

- Regular sets are a class of languages central to much of the language theory
- We will see several methods for specifying these languages
- Regular expressions
- Right-linear grammars
- Deterministic finite-state automata
- Non deterministic finite-state automata
$\Rightarrow$ All this formalisms are in fact equivalent


## Regular sets - definition

- Let $\Sigma$ be a finite alphabet. A regular set over $\Sigma$ is defined recursively as follows:
- the empty language $\varnothing$ is a regular language.
- the empty string language $\{\varepsilon\}$ is a regular language.
- For each $a \in \Sigma$, the singleton language $\{a\}$ is a regular language.
- If $A$ and $B$ are regular languages, then $A \cup B$ (union), $A B$ (concatenation), and $A^{*}$ (Kleene star) are regular languages.
- No other languages over $\Sigma$ are regular.

A simple example of a language that is not regular is $\left\{a^{n} b^{n} \mid n \geq 0\right\}$

## Regular expressions

- Regular expressions over $\Sigma$ and the regular sets they denote are defined recursively as follows:
- $\varnothing$ is a regular expression denoting the empty set
- $\varepsilon$ is a regexpr denoting the regular set $\{\varepsilon\}$
- a in $\Sigma$ is a regexp denoting $\{a\}$
- If $p$ and $q$ are regexp denoting $P$ and $Q$, then
- $(p \mid q)$ is a regexp denoting $P \cup Q$
- (pq) is a regexp denoting PQ
- $(\mathrm{P})^{*}$ is regexp denoting $\mathrm{P}^{*}$
- Nothing else is a regular expression


## Examples

- The finite languages, i.e. those containing only a finite number of words
$\Rightarrow$ These are obviously regular as one can create a regular expression that is the union of every word in the language, and thus are regular
- 01 denoting $\{01\}$
- 0* denoting $\{0\}^{*}$
- (0|1)* denoting $\{0,1\}^{*}$
- (0|1)*011 denoting all strings of 0's and 1's ending in 011


## Examples (cont.)

- Given the alphabet $\Sigma=\{\mathrm{a}, \mathrm{b}\}$ :
$\rightarrow$ ba* - all the strings that begin with ab followed only by a's
$\rightarrow$ a*ba*ba* - strings that contain exactly two b's
$\rightarrow(\mathrm{a} \mid \mathrm{b})^{*}$ - all the strings on $\Sigma$
$\rightarrow(a \mid b)^{*}(a a \mid b b)(a \mid b)^{*}$ - all the string on $\Sigma$ that contain either two consecutive a's or two consecutive b's
$\rightarrow\left[a a|b b|(a b \mid b a)(a a \mid b b)^{*}(a b \mid b a)\right]^{*}$ - strings that contain an even number of a 's and an even number of b's
$\rightarrow(\mathrm{b} \mid \mathrm{abb})^{*}$ - strings on $\Sigma$ in which an a is followed immediately by at least two b's


## Basic algebraic properties

- Let $\alpha, \beta$, and $\gamma$ regular expressions
- $\alpha|\beta=\beta| \alpha$
- $\alpha|(\beta \mid \gamma)=(\alpha \mid \beta)| \gamma \quad \alpha(\beta \gamma)=(\alpha \beta) \gamma$
- $\varnothing^{*}=\varepsilon$
- $\alpha(\beta \mid \gamma)=\alpha \beta|\alpha \gamma \quad(\alpha \mid \beta) \gamma=\alpha \gamma| \beta \gamma$
- $\alpha \varepsilon=\varepsilon \alpha=\alpha$
- $\alpha^{*}=\alpha \mid \alpha^{*} \quad\left(\alpha^{*}\right)^{*}=\alpha^{*}$
- $\alpha|\alpha=\alpha \quad \alpha| \varnothing=\alpha$
- All these properties are demonstrable by reasoning on the respective denoted sets


## Finite State Automata

- We have seen some ways to define the class of the regular sets:
- The regular sets are those sets defined by regular expressions
- The regular sets are the languages generated by right-linear grammar
- We will see another way: regular sets defined by Finite Automata


## Finite State Automata

- A finite-state automaton consists only of an input tape and a finite control
- A finite control means that the device that can be in one among a finite number of states
- In certain conditions, it can switch to another state $=>$ this is called a transition
- Allowable input symbols
- Initial and final states
- If the automaton is in a final state when it stops working, it is said to accept its input


## FSA - transitions

- A state transition function that, given the "current" state and the "current" input symbol, returns all possible next states
- In principle, this device is non-deterministic: the device goes in all its next states, such as it replicates itself
- The device accepts the inputs if any of its parallel existences reaches an accepting state
$\xrightarrow[b]{\text { intial state }} q_{i} \rightarrow q_{k} d \xrightarrow[\text { farmals state }]{ }$
 $c$


## FSA - definitions

- A non-deterministic finite state automaton is a 5-tuple $M=\left(Q, \Sigma, \delta, q_{0}, F\right)$, such that

1. $Q$ is a finite state of states
2. $\Sigma$ is a finite set of allowable input symbols
3. $\delta$ is a state transition function, i.e. a mapping from $Q \times \Sigma$ to $\mathcal{P}(Q)$ that defines the finite state control
4. $q_{0}$ in $Q$ is the initial state
5. $F \subseteq Q$ is the set of final states

## FSA - definitions

- To determine the future behavior of a FSA, all we need to know is its configuration
- The current state of the finite control
- The string symbol on the input tape (= the symbol under the input head, followed by all symbols on the right)
- A move is represented as

$$
(q, a w) \rightarrow\left(q^{\prime}, w\right)
$$

means:

- The automaton is in the current state q
- The input head is scanning the symbol a
- The automaton may change its state to $\mathrm{q}^{\prime}$ and shift the input head on the right


## FSA - example

- Let $M=(\{p, q, r\},\{0,1\}, \delta, p,\{r\})$ a FSA where $\delta$ is defined as:

|  | Input |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\delta$ | 0 | 1 |  |
| State | $p$ | \{q\} | \{p\} | p: Two consecutive 0 ' have not appeared yet |
|  | $q$ | \{r\} | \{p\} | q: Two consecutive $0^{\prime}$ have not appeared, but the previous symbol was a 0 |
|  | $r$ | \{r\} | \{r\} | r: Two consecutive $0^{\prime}$ ' have appeared |

- M accepts string of 0's and 1's that contains two consecutive 0's

On input 01001, we have:
$(p, 01001) \rightarrow(q, 1001) \rightarrow(p, 001) \rightarrow(q, 01) \rightarrow(r, 1) \rightarrow(r, \varepsilon)$

## FSA - non-deterministic case

- Design a non-deterministic FSA to accept the strings
- in the alphabet $\{1,2,3\}$,
- and such that the last symbol in the input string also appear previously in the string
- e.g. 121 is accepted, 31312 not
- We will need some state, an initial state $q_{0}$ (nothing has been recognized), $q_{1} q_{2} q_{3}$ some guessing has been made, and a final $q_{f}$


## FSA - non-deterministic case (2)

- More formally:
$M=\left(\left\{q_{0}, q_{1}, q_{2}, q_{3}, q_{f}\right\},\{1,2,3\}, \delta, q_{0},\left\{q_{f}\right\}\right)$
Input

|  | $\delta$ | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| State | $q_{0}$ | $\left\{q_{0}, q_{1}\right\}$ | $\left\{q_{0}, q_{2}\right\}$ | $\left\{q_{0}, q_{3}\right\}$ |
|  | $q_{1}$ | $\left\{q_{1}, q_{f}\right\}$ | $\left\{q_{1}\right\}$ | $\left\{q_{1}\right\}$ |
|  | $q_{2}$ | $\left\{q_{2}\right\}$ | $\left\{q_{2,} q_{f}\right\}$ | $\left\{q_{2}\right\}$ |
|  | $q_{3}$ | $\left\{q_{3}\right\}$ | $\left\{q_{3}\right\}$ | $\left\{q_{31} q_{f}\right\}$ |
|  | $q_{f}$ | $\varnothing$ | $\varnothing$ | $\varnothing$ |

## FSA - non-deterministic case (3)

- On input 12321, the configurations will be $\left(q_{0}, 12321\right) \rightarrow\left(q_{0}, 2321\right)-$


Since $\left(q_{0}, 12321\right) \xrightarrow{*}\left(q_{f}, e\right)$, the string 12321 is in $L(M)$

## FSA - transition graph

- It is often convenient to have a graph representation of finite automata
- E.g.: $\quad M=(\{p, q, r\},\{0,1\}, \delta, p,\{r\})$ with

|  | Input |  |  |
| :---: | :---: | :---: | :---: |
|  | $\delta$ | 0 | 1 |
| State | $p$ | $\{q\}$ | $\{p\}$ |
|  | $q$ | $\{r\}$ | $\{p\}$ |
|  | $r$ | $\{r\}$ | $\{r\}$ |

can be represented as



## FSA and non deterministic FSA

- There is an equivalence to deterministic and non-deterministic FSA:
- Theorem:

If $L=L(M)$ for some non-deterministic FSA M, then there is a $M^{\prime}$ such that $L=L\left(M^{\prime}\right)$
$\Rightarrow$ In the case of finite state automata, determinism and non-determinism have the same expressive power

## Non-deterministic $\rightarrow$ deterministic transformation

- Theorem:

If $L=L(M)$ for some non-deterministic FSA
$M$, then there is a $M^{\prime}$ such that $L=L\left(M^{\prime}\right)$

- $M=\left(Q, \Sigma, \delta, q_{0}, F\right)$.

We construct $M^{\prime}=\left(Q^{\prime}, \Sigma, \delta^{\prime}, q^{\prime}, F^{\prime}\right)$, such that

1) $Q^{\prime}=\mathcal{P}(Q)$, i.e. the powersets (sets of states) of $M$
2) $q_{0}^{\prime}=\left\{q_{0}\right\}$
3) $\mathrm{F}^{\prime}$ consists of all subsets S of Q s.t. $\mathrm{S} \cap \mathrm{F} \neq \varnothing$
4) For all $S \subseteq Q$, $\delta^{\prime}(S, a)=S^{\prime}$, where $S^{\prime}=\{p \mid \delta(q, a)$ contains $p$ for some $q$ in $S\}$

## N-FSA to D-FSA in practice

- Given an N-FSA, we can construct an equivalent D-FSA
- States in the D-FSA correspond to the powersets of states in the N-FSA
- Straightforward way of computing D-FSA:
- Create a list of all powersets of states in N-FSA
- Add transitions according to those in the original N-FSA
- Remove any states which cannot be reached


## N-FSA to D-FSA in practice

Example:


We recall that $|\mathcal{P}(X)|=2^{|x|}$
Powersets are

$$
\varnothing, q_{0}, q_{1}, q_{2},\left\{q_{0}, q_{1}\right\},\left\{q_{0}, q_{2}\right\},\left\{q_{1}, q_{2}\right\},\left\{q_{0}, q_{1}, q_{2}\right\}
$$

## N-FSA to D-FSA




## N-FSA to D-FSA



Highlighted states can't be reached (there are no transitions to them) or they are sink (lead to no acceptance states). So we can eliminate them.


## N-FSA to D-FSA

- Considering all powersets can lead to states in the D-FSA which cannot be reached and they have to be removed
- The number of powersets immediately becomes very large (an N-FSA with 20 states would have $2^{20}=1.048 .576$ states!)
- We don't really need to consider all powersets: only those to which there are transitions in the original N-FSA have to be considered


## Transformation Regexp <-> FSA

- Theorem (Kleene):

To each regular expression there corresponds a FSA and to each FSA there corresponds a regular expression

- We will give an algorithm to switch from these two objects


## Transformation Regexp <-> FSA

- We can observe that ( $\alpha, \beta, \alpha_{i}$ are regular expressions):



## Node elimination

- Suppose we want to eliminate the node $\mathrm{q}_{2}$ from the graph:



## FSA -> Regexp

- An example to transform a FSA into a regexp



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## Regexp -> FSA

- Let us consider the regexp
( $a \mid b)^{*}(a a \mid b b)(a \mid b)^{*}$



## Equivalence of FSA's

- Theorem (Moore):

There exists an algorithm, to determine if two FSA's on an alphabet $\Sigma$ are equivalent

- An algorithm:
- A and $A^{\prime}$ two FSA's on $\Sigma=\{0,1\}$.
- We rename the nodes, to have different labels in $A$ and $A^{\prime}$
- We build a table of comparisons, with three columns, in this way:


## Equivalence of FSA's (cont.)



## Equivalence of FSA's (cont.)

- If in the table, we get to a pair ( $v, v^{\prime}$ ), where $v$ is an acceptance state and $v^{\prime}$ not, $\Rightarrow>A$ and $A^{\prime}$ are not equivalent
- If we get to an end, i.e. there is no pair in the columns 2 and 3 that is not present in column 1, $=>$ the $A$ and $A^{\prime}$ are equivalent


# Automata theory: formal languages and formal grammars 

| Chomsky hierarchy | Grammars | Languages | Minimal automaton |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Type-0 | Unrestricted | Recursively enumerable | Turing machine |
| n/a | (no common name) | Recursive | Decider |
| Type-1 | Context-sensitive | Context-sensitive | Linear-bounded |
| n/a | Indexed | Indexed | Nested stack |
| n/a | Tree-adjoining | Mildly context-sensitive | Thread |
| Type-2 | Context-free | Context-free | Nondeterministic pushdown |
| n/a | Deterministic | Deterministic | Deterministic pushdown |
|  | context-free | context-free |  |
| Type-3 | Regular | Regular | Finite state |

Each category of languages or grammars is a proper subset of the category directly above it.

## Tokenization

- Wordforms, inflected words as it appears in the corpus
- e.g. cat and cats are treated as two separated words
- Lemma
- We might want to treat cat and cats as instances of a single lemma "cat"
- Types: distinct words in a corpus, i.e. the size of the vocabulary
- Tokens: the total number of running words
- The Brown corpus contains 1 million wordform tokens, that is 61,803 wordform types, that is 37,851 lemma types


## Tokenization

- Types and tokens
- The following sentence taken from the Brown corpus:
"They picnicked by the pool, then lay back on the grass and looked at the stars"
- has 16 word tokens and 14 word types (not counting punctuation)


## Tokenization

- A simple automaton for the recognition of the tokens


A delimiter can be any character that is not a letter or a digit

## Regexp in the "real world"

- It is worth noting that many real-world "regular expression" engines implement features that cannot be expressed in the regular expression algebra
- Some examples:
- grep, Unix command line
- AWK, Unix command line, progr. language
- Emacs, a powerful editor
- Perl, a programming language
- Pregexp package, in Scheme


## Grep - a Unix command

- grep, egrep, fgrep - print lines matching a pattern [egrep = grep -e]
- SYNOPSIS
- grep [options] PATTERN [FILE...]
- grep [options] [-e PATTERN | -f FILE] [FILE...]
- grep searches the named input FILEs (or standard input if no files are named, or the file name - is given) for lines containing a match to the given PATTERN. By default, grep prints the matching lines
- egrep is used when the pattern is a regular expression


## Grep - a Unix command

- grep fish fortunes
- A woman without a man is like a fish without a bicycle.
- No one can feel as helpless as the owner of a sick goldfish.
- Time is about the stream I go a-fishing in.
- fgrep inst /etc/passwd
- glenn:*:301:300:Glenn Stafford-instructor:/u/glenn:/bin/ksh
- institution:*:301:300:Database Acct:/u/db:/bin/ksh


## grep - other examples

- grep -i apple fruitlist.txt
$\Rightarrow$ returns all lines with the words 'apple', 'Apple', 'apPLE', or any other mixing of capital and lower case
- grep -r 'hello'/home/gigi
$\Rightarrow$ searches for 'hello' in all files under the directory '/home/gigi'


## Grep - regular expressions

## A regular expression may be followed by one of several repetition operators:

- The period . matches any single character.
- ? The preceding item is optional and will be matched at most once.
- The preceding item will be matched zero or more times.
- [^] Match any one character except those enclosed in [], as in [^0-9].
-     + The preceding item will be matched one or more times.
- $\{n\}$ The preceding item is matched exactly $n$ times.
- $\{n$,$\} The preceding item is matched n$ or more times.
- $\{n, m\}$ The preceding item is matched at least $n$ times, but not more than $m$ times.
- Two regular expressions may be concatenated;
- Two regular expressions may be joined by the infix operator | ; the resulting regular expression matches any subexpression


## grep - examples

- An example is

```
            (hurrah ) {2,3}
```

which matches
hurrah hurrah
as well as
hurrah hurrah hurrah

- A more complex example combines alternation and grouping with a quantifier:
(hurrah |yahoo ) $\{2,3\}$

That gives twelve possible combinations, including for example
hurrah yahoo
and
yahoo hurrah yahoo

## grep - examples

egrep '((the|a) (big( red)?|small(yellow)?) (car|bike))' car.txt
the big red car
a small bike
the small yellow car
a big red bike

## Anchors

- Using ^ and \$, you can force a regexp to match only at the beginning ^ or at the end \$ of a line
- E.g. ${ }^{\wedge}$ cat matches only those lines that start with cat, and cat\$ matches only those lines that end with cat
- \< and \> are start-of-word, end-of-word anchors
- E.g. $\backslash<c a t \backslash>$ looks for only the word cat


## Anchors

```
grep 'cat' cats.txt grep '\<cat' cats.txt
cat
cattle
    cat
    cattle
catalog
    catalog
    scrawny cat scrawny cat
    vacation
    wildcat
```

```
grep '\<cat\> cats.txt
                cat
                scrawny cat
```


## Anchors

- These word boundaries are not supported in all regexp engines implementations
- Some implementations (inluding per) offer is-a-word-boundary and not-a-word-boundary
- $\backslash b$ and $\backslash B$ respectively

```
grep '\bcat\b' cats.txt
    cat
    scrawny cat
```


## Character classes

- The [...] construct indicates the presence of one of the enclosed characters
- E.g. c[ao]ke matches cake and coke
- [0123456789abcdefABCDEF] is also written as [0-9a-fA-F]
- [ ^...] means a 'negated’ character set
- E.g. [ ${ }^{\wedge} 0-9$ ] means any character except digits


## Dot

- The dot . is a special character and matches any character
- E.g. th.s matches this, thus, thgs, th@s, ...
- When you have to match a dot, you need to 'escaped' it => l.
- E.g. to match the IP address 74.6.7.121 all three dots need to be escaped $74 \backslash .6 \backslash .7 \backslash .121$


## Quantifiers

- Using quantifiers, it is possible to specify how often a pattern may or must be repeated
- The general form is \{min, max \}
- Examples:
- bo 1,2$\} \mathrm{k}$ matches both book and bok
- [aeiou] $\{3,5\}$ matches any sequence of three to five vowels
- finds $\{0,1\}$ matches find and $f$ inds
- finds $\{0,1\}=$ finds?
- $\uparrow\{80,80\}$ \$
$\rightarrow$ matches lines of exactly eighty dash


## Alternation and grouping

- The meta character | means or
- ^(From|Subject|Date):
$\Leftrightarrow$ filters e-mail headers
- (...) has the function of grouping for quantifiers
- (hurrah ) $\{2,3\}$ matches hurrah hurrah hurrah
- (hurrah | yahoo ) \{2,3\} matches hurrah yahoo or yahoo hurrah yahoo etc.


## Backreferencing

- Grouping has a very useful side-effect
- Certain regexp implementations remember the matched text in a grouping
- E.g. searching for double words in a text, like ... when when
- $([a-z A-z]+)\{1\}$ the $\backslash 1$ is called avorackreference to the first group, in this case $([a-z A-Z]+)$
- maybe better $([a-z A-Z]+) \backslash 1 \backslash>$
- The max number of backreferences is limited to nine in most regexp implementations


## grep - regular expressions

- How to express palindromes in a regular expression?
- It can be done by using the back references, for example a palindrome of 5 characters can be written in
- grep -e '<br>(.<br>)<br>(.<br>).\2\1' file
- It matches the word "radar" or "civic".



## Emacs and regexp

- Emacs is a powerful text editor
- Let us give a look at its regexp facilities
- An interactive command "replace-regexp"
- Transform every line in a file (e.g. /etc/passwd) that matches
- ${ }^{\wedge} \backslash\left([\wedge:]^{*} \backslash\right):[\wedge:] *: \backslash([0-9] * \backslash):[0-9]^{*}: \backslash([\wedge:] * \backslash): . * \$$
- into
- Login $\{\backslash 1\}$ Full Name $\{\backslash 3\}$ UID $\{\backslash 2\}$
- Ex. It matches the line
- mysql:*:74:74:MySQL Server:/var/empty:/usr/bin/false
- $\wedge([\wedge:] * \backslash):[\wedge:] *: \backslash([0-9] * \backslash):[0-9] *: \backslash([\wedge:] * \backslash): . * \$$


## Exercise

- ALPHABET: a b c
- Write a regular expression for the language of all strings over the alphabet $\{a, b, c\}$ that start with character a

Solution: $\quad a(a|b| c)^{*}$

## Exercise

- ALPHABET: a b c
- Write a regular expression for the language of all strings over the alphabet $\{a, b, c\}$ that start and end with the character a

SOLUTION: $\quad a(a|b| c) * a \mid a$

## Exercise

- ALPHABET: a b c
- Write a regular expression for the language of all strings over the alphabet $\{a, b, c\}$ that start with character a, but do not end with character a

SOLUTION: $\quad a(\mathrm{a}|\mathrm{b}| \mathrm{c})^{*}(\mathrm{~b} \mid \mathrm{c})$

## Exercise

- ALPHABET: a b c
- Give a regular expression over $\{a, b, c\}$ where a must appear in blocks of even length

$$
\text { SOLUTION: } \quad(\mathrm{aa}|\mathrm{~b}| \mathrm{c})^{*}
$$

## Exercise

- ALPHABET: 01 x
- Write a regular expression for the language of all strings over the alphabet $\{0,1, x\}$ that contain at least one $x$

SOLUTION: (0|1)*x(0|1|x)*

## Different syntax in the real engines

- The practical regexp engines use different syntax for writing the regular expressions
- Simple matching
- POSIX basic
- POSIX extended
- Emacs
- Grep
- GNU regex
- Java
- Perl
- Ruby
- ...
- Mainly small differences, but before using a tool you have to read the manual

