5.

Reasoning with Horn Clauses

Horn clauses

Clauses are used two ways:

- as disjunctions: (rain ∨ sleet)
- as implications: (¬child ∨ ¬male ∨ boy)

Here focus on 2nd use

Horn clause = at most one +ve literal in clause

- positive / definite clause = exactly one +ve literal
  e.g. $[\neg p_1, \neg p_2, ..., \neg p_n, q]$
- negative clause = no +ve literals
  e.g. $[\neg p_1, \neg p_2, ..., \neg p_n]$ and also [ ]

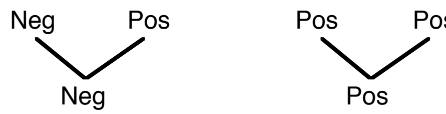
Note: $[\neg p_1, \neg p_2, ..., \neg p_n, q]$ is a representation for
$(\neg p_1 \vee \neg p_2 \vee ... \vee \neg p_n \vee q)$ or $[(p_1 \wedge p_2 \wedge ... \wedge p_n) \supset q]$

so can read as: If p_1 and p_2 and ... and p_n then q

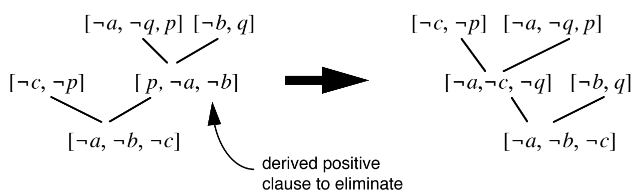
and write as: $p_1 \wedge p_2 \wedge ... \wedge p_n \Rightarrow q$ or $q \Leftarrow p_1 \wedge p_2 \wedge ... \wedge p_n$

Resolution with Horn clauses

Only two possibilities:



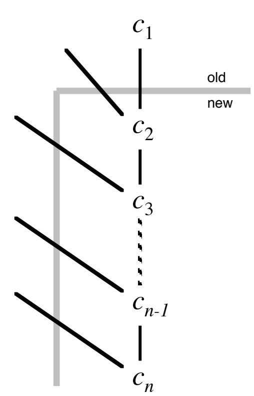
It is possible to rearrange derivations of negative clauses so that all new derived clauses are negative



Further restricting resolution

Can also change derivations such that each derived clause is a resolvent of the previous derived one (negative) and some positive clause in the original set of clauses

- Since each derived clause is negative, one parent must be positive (and so from original set) and one parent must be negative.
- Chain backwards from the final negative clause until both parents are from the original set of clauses
- Eliminate all other clauses not on this direct path



This is a recurring pattern in derivations

- See previously:
  - example 1, example 3, arithmetic example
- But not:
  - example 2, the 3 block example

SLD Resolution

An <u>SLD-derivation</u> of a clause c from a set of clauses S is a sequence of clause $c_1, c_2, \ldots c_n$ such that $c_n = c$, and

1. $c_1 \in S$
2. c_{i+1} is a resolvent of c_i and a clause in S

Write: $S \overset{\text{SLD}}{\rightarrow} c$

SLD means S(elected) literals
L(inear) form
D(efinite) clauses

Note: SLD derivation is just a special form of derivation and where we leave out the elements of S (except c_1)

In general, cannot restrict ourselves to just using SLD-Resolution

Proof: $S = \{[p, q], [p, \neg q], [\neg p, q] [\neg p, \neg q]\}$. Then $S \rightarrow []$.

Need to resolve some [ρ] and [$\overline{\rho}$] to get [].
But S does not contain any unit clauses.

So will need to derive both [ρ] and [$\overline{\rho}$] and then resolve them together.

Completeness of SLD

However, for Horn clauses, we can restrict ourselves to SLD-Resolution

Theorem: SLD-Resolution is refutation complete for Horn clauses: $H \rightarrow []$ iff $H \overset{\text{SLD}}{\rightarrow} []$

So: H is unsatisfiable iff $H \overset{\text{SLD}}{\rightarrow} []$

This will considerably simplify the search for derivations

Note: in Horn version of SLD-Resolution, each clause in the $c_1, c_2, \ldots, c_n$, will be negative

So clauses H must contain at least one negative clause, c_1 and this will be the only negative clause of H used.

Typical case:

– KB is a collection of positive Horn clauses
– Negation of query is the negative clause

Example 1 (again)

SLD derivation

alternate representation

KB

FirstGrade
FirstGrade ⊃ Child
Child ∧ Male ⊃ Boy
Kindergarten ⊃ Child
Child ∧ Female ⊃ Girl
Female

[¬Girl]

|

[¬Child, ¬Female]

|

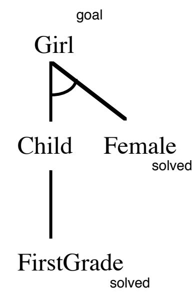
[¬Child]

|

[¬FirstGrade]

|

[]



A goal tree whose nodes are atoms, whose root is the atom to prove, and whose leaves are in the KB

Show KB ∪ {¬Girl} unsatisfiable

Prolog

Horn clauses form the basis of Prolog

Append(nil,y,y)

Append(x,y,z) ⇒ Append(cons(w,x),y,cons(w,z))

What is the result of appending [c] to the list [a,b] ?

Append(cons(a,cons(b,nil)), cons(c,nil), u) goal

| u / cons(a,u')

Append(cons(b,nil), cons(c,nil), u')

| u' / cons(b,u'')

Append(nil, cons(c,nil), u'')

solved: u'' / cons(c,nil)

So goal succeeds with u = cons(a,cons(b,cons(c,nil)))
that is: Append([a b],[c],[a b c])

With SLD derivation, can always extract answer from proof

$H \models \exists x\, \alpha(x)$

iff

for some term t, $H \models \alpha(t)$

Different answers can be found by finding other derivations

Back-chaining procedure

Solve[$q_1, q_2, ..., q_n$] = /* to establish conjunction of q_i */

If n=0 then return **YES**; /* empty clause detected */

For each $d \in$ KB do

If $d = [q_1, \neg p_1, \neg p_2, ..., \neg p_m]$ /* match first q */

and /* replace q by -ve lits */

Solve[$p_1, p_2, ..., p_m, q_2, ..., q_n$] /* recursively */

then return **YES**

end for; /* can't find a clause to eliminate q */

Return **NO**

Depth-first, left-right, back-chaining

- depth-first because attempt p_i before trying q_i
- left-right because try q_i in order, 1,2, 3, ...
- back-chaining because search from goal q to facts in KB p

This is the execution strategy of Prolog

First-order case requires unification *etc.*

Problems with back-chaining

Can go into infinite loop

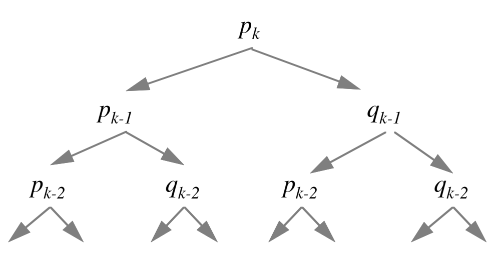
tautologous clause: $[p\ ,\ \neg p]$ (corresponds to Prolog program with `p :- p`).

Previous back-chaining algorithm is inefficient

Example: Consider 2n atoms, $p_0, ..., p_{n-1}, q_0, ..., q_{n-1}$ and $4n$-4 clauses

$$(p_{i-1} \Rightarrow p_i),\ (q_{i-1} \Rightarrow p_i),\ (p_{i-1} \Rightarrow q_i),\ (q_{i-1} \Rightarrow q_i).$$

With goal p_k the execution tree is like this



Solve[p_k] eventually fails after 2^k steps!

Is this problem inherent in Horn clauses?

Forward-chaining

Simple procedure to determine if Horn KB $\models q$.

main idea: mark atoms as solved

> 1. If q is marked as solved, then return **YES**
> 2. Is there a $\{p_1, \neg p_2, \ldots, \neg p_n\} \in$ KB such that $p_2, \ldots, p_n$ are marked as solved, but the positive lit p_1 is not marked as solved?
>
>   no: return **NO**
>
>   yes: mark p_1 as solved, and go to 1.

FirstGrade example:

Marks: FirstGrade, Child, Female, Girl then done!

Note: FirstGrade gets marked since all the negative atoms in the clause (none) are marked

Observe:

- only letters in KB can be marked, so at most a linear number of iterations
- not goal-directed, so not always desirable
- a similar procedure with better data structures will run in *linear* time overall

First-order undecidability

Even with just Horn clauses, in the first-order case we still have the possibility of generating an infinite branch of resolvents.

KB:

$\text{LessThan}(\text{succ}(x),y) \Rightarrow \text{LessThan}(x,y)$

Query:

$\text{LessThan}(\text{zero},\text{zero})$

As with full Resolution, there is no way to detect when this will happen

$$\begin{array}{c}
[\neg \text{LessThan}(0,0)] \\
\downarrow \; x/0,\, y/0 \\
[\neg \text{LessThan}(1,0)] \\
\downarrow \; x/1,\, y/0 \\
[\neg \text{LessThan}(2,0)] \\
\downarrow \; x/2,\, y/0 \\
\ldots
\end{array}$$

There is no procedure that will test for the satisfiability of first-order Horn clauses

the question is undecidable

As with non-Horn clauses, the best that we can do is to give control of the deduction to the *user*

to some extent this is what is done in Prolog, but we will see more in "Procedural Control"

6.

Procedural Control of Reasoning

Declarative / procedural

Theorem proving (like resolution) is a general domain-independent method of reasoning

Does not require the user to know how knowledge will be used

will try all logically permissible uses

Sometimes we have ideas about how to use knowledge, how to search for derivations

do not want to use arbitrary or stupid order

Want to communicate to theorem-proving procedure some *guidance* based on properties of the domain

- perhaps specific method to use
- perhaps merely method to avoid

Example: directional connectives

In general: control of reasoning

DB + rules

Can often separate (Horn) clauses into two components:

Example:

MotherOf(jane,billy)
FatherOf(john,billy)
FatherOf(sam, john)
...

a database of facts
- basic facts of the domain
- usually ground atomic wffs

ParentOf(x,y) $\Leftarrow$ MotherOf(x,y)
ParentOf(x,y) $\Leftarrow$ FatherOf(x,y)
ChildOf(x,y) $\Leftarrow$ ParentOf(y,x)
AncestorOf(x,y) $\Leftarrow$ ...
...

collection of rules
- extends the predicate vocabulary
- usually universally quantified conditionals

Both retrieved by unification matching

Control issue: how to use the rules

Rule formulation

Consider AncestorOf in terms of ParentOf

Three logically equivalent versions:

1. AncestorOf(x,y) $\Leftarrow$ ParentOf(x,y)
   AncestorOf(x,y) $\Leftarrow$ ParentOf(x,z) $\wedge$ AncestorOf(z,y)
2. AncestorOf(x,y) $\Leftarrow$ ParentOf(x,y)
   AncestorOf(x,y) $\Leftarrow$ ParentOf(z,y) $\wedge$ AncestorOf(x,z)
3. AncestorOf(x,y) $\Leftarrow$ ParentOf(x,y)
   AncestorOf(x,y) $\Leftarrow$ AncestorOf(x,z) $\wedge$ AncestorOf(z,y)

Back-chaining goal of AncestorOf(sam,sue) will ultimately reduce to set of ParentOf(–,–) goals

1. get ParentOf(sam,z): find child of Sam searching *downwards*
2. get ParentOf(z,sue): find parent of Sue searching *upwards*
3. get ParentOf(–,–): find parent relations searching *in both directions*

Search strategies are not equivalent

if more than 2 children per parent, (2) is best

Algorithm design

Example: Fibonacci numbers

1, 1, 2, 3, 5, 8, 13, 21, ...

Version 1:

$\mathrm{Fibo}(0, 1)$

$\mathrm{Fibo}(1, 1)$

$\mathrm{Fibo}(\mathrm{s}(\mathrm{s}(n)), x) \Leftarrow \mathrm{Fibo}(n, y) \wedge \mathrm{Fibo}(\mathrm{s}(n), z) \wedge \mathrm{Plus}(y, z, x)$

Requires *exponential* number of Plus subgoals

Version 2:

$\mathrm{Fibo}(n, x) \Leftarrow \mathrm{F}(n, 1, 0, x)$

$\mathrm{F}(0, c, p, c)$

$\mathrm{F}(\mathrm{s}(n), c, p, x) \Leftarrow \mathrm{Plus}(p, c, s) \wedge \mathrm{F}(n, s, c, x)$

Requires only *linear* number of Plus subgoals

Ordering goals

Example:

$\mathrm{AmericanCousinOf}(x,y) \Leftarrow \mathrm{American}(x) \wedge \mathrm{CousinOf}(x,y)$

In back-chaining, can try to solve either subgoal first

Not much difference for AmericanCousinOf(fred, sally), but big difference for AmericanCousinOf(x, sally)

1. find an American and then check to see if she is a cousin of Sally
2. find a cousin of Sally and then check to see if she is an American

So want to be able to order goals

better to <u>generate</u> cousins and <u>test</u> for American

In Prolog: order clauses, and literals in them

Notation: G :- $G_1, G_2, ..., G_n$ stands for
$G \Leftarrow G_1 \wedge G_2 \wedge ... \wedge G_n$
but goals are attempted in presented order

Commit

Need to allow for backtracking in goals

AmericanCousinOf(x,y) :- CousinOf(x,y), American(x)

for goal AmericanCousinOf(x,sally), may need to try to solve the goal American(x) for many values of x

But sometimes, given clause of the form

G :- *T*, *S*

goal *T* is needed only as a *test* for the applicability of subgoal *S*

- if *T* succeeds, commit to *S* as the *only* way of achieving goal *G*.
- if *S* fails, then *G* is considered to have failed
  - do not look for other ways of solving *T*
  - do not look for other clauses with *G* as head

In Prolog: use of cut symbol

Notation: G :- T_1, T_2, ..., T_m, !, G_1, G_2, ..., G_n

attempt goals in order, but if all T_i succeed, then commit to G_i

If-then-else

Sometimes inconvenient to separate clauses in terms of unification:

G(zero, –) :- *method 1*
G(succ(n), –) :- *method 2*

For example, may split based on computed property:

Expt(a, n, x) :- Even(n), ... (*what to do when* n *is even*)
Expt(a, n, x) :- Even(s(n)), ... (*what to do when* n *is odd*)

want: check for even numbers only once

Solution: use ! to do if-then-else

G :- P, !, Q.
G :- R.

To achieve G: if P then use Q else use R

Example:

Expt(a, n, x) :- $n = 0$, !, $x = 1$.
Expt(a, n, x) :- Even(n), !, (*for even* n)
Expt(a, n, x) :- (*for odd* n)

Note: it would be correct to write

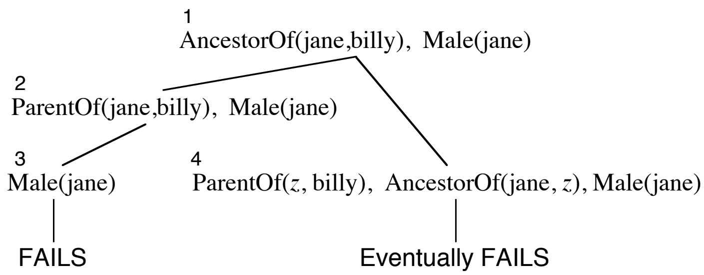
Expt(a, 0, x) :- !, $x = 1$.

but not

Expt(a, 0, 1) :- !.

Controlling backtracking

Consider solving a goal like



So goal should really be: AncestorOf(jane,billy), !, Male(jane)

Similarly:

Member(x,l) $\Leftarrow$ FirstElement(x,l)
Member(x,l) $\Leftarrow$ Rest(l,l') $\wedge$ Member(x,l')

If only to be used for testing, want

Member(x,l) :- FirstElement(x,l), !, .

On failure, do not try to find another x later in the rest of the list

Negation as failure

Procedurally: we can distinguish between the following:

can solve goal $\neg G$ *vs.* cannot solve goal G

Use **not**(G) to mean the goal that succeeds if G fails, and fails if G succeeds

Roughly: **not**(G) :- G, !, fail. /* fail if G succeeds */
not(G). /* otherwise succeed */

Only terminates when failure is *finite* (no more resolvents)

Useful when DB + rules is complete

NoChildren(x) :- **not**(ParentOf(x,y))

or when method already exists for complement

Composite(n) :- $n > 1$, **not**(PrimeNum(n))

Declaratively: same reading as ¬, but not when *new* variables in G

[**not**(ParentOf(x,y)) $\supset$ NoChildren(x)] ✔
vs. [$\neg$ParentOf(x,y) $\supset$ NoChildren(x)] ✘

Dynamic DB

Sometimes useful to think of DB as a snapshot of the world that can be changed dynamically

assertions and deletions to the DB

then useful to consider 3 procedural interpretations for rules like

$$\text{ParentOf}(x,y) \Leftarrow \text{MotherOf}(x,y)$$

1. If-needed: Whenever have a goal matching ParentOf(x,y), can solve it by solving MotherOf(x,y)

   ordinary back-chaining, as in Prolog

2. If-added: Whenever something matching MotherOf(x,y) is added to the DB, also add ParentOf(x,y)

   forward-chaining

3. If-removed: Whenever something matching ParentOf(x,y) is removed from the DB, also remove MotherOf(x,y), if this was the reason

   keeping track of dependencies in DB

Interpretations (2) and (3) suggest demons

procedures that monitor DB and fire when certain conditions are met

The Planner language

Main ideas:

1. DB of facts

   (Mother susan john) (Person john)

2. If-needed, if-added, if-removed procedures consisting of
   - body: program to execute
   - pattern for invocation (Mother x y)

3. Each program statement can succeed or fail
   - **(goal** *p***)**, **(assert** *p***)**, **(erase** *p***)**,
   - **(and** *s* ... *s***)**, statements with backtracking
   - **(not** *s***)**, negation as failure
   - **(for** *p* *s***)**, do *s* for every way *p* succeeds
   - **(finalize** *s***)**, like cut
   - a lot more, including all of Lisp

Shift from proving conditions to making conditions hold!

examples:

```
(proc if-needed (cleartable)
  (for (on x table)
    (and (erase (on x table)) (goal (putaway x)))))
(proc if-removed (on x y) (print x " is no longer on " y))
```