Prolog Core Concepts and Notation

Yves Lespérance
Adapted from Peter Roosen-Runge
Readings: C & M Ch 1, 2, 3.1-3.3, 8

declarative/logic programming

- idea: write a program that is a logical theory about some domain and then query it
- most well known instance is Prolog
- core constructs, terms and statements, are inherited from first order logic

terms

- Prolog statements express relationships among *terms*
- terms are (a generalization) of the same notion in first order logic, i.e. a constant, a variable, or a function applied to some argument terms
- E.g. john, john_smith, X, Node, _person, fatherOf(paul), date(25,10,2005)
- fatherOf and date are functors; date has arity 3; it takes 3 arguments

terms

- variables begin with upper-case letter or _
- constants and functors (symbols) begin with lower-case
- terms denote objects
- compound terms are called structures
- E.g. course(complexity,time(Monday,9,11),lecturer(patrick,dymond),location(CSE,3311))
- used to represent complex data
- terms (usually) have a tree structure

facts

- *facts* are like atomic formulas in first order logic.
- syntax is same as terms, but ending with a period.
- e.g. fatherOf(paul,henry).
  mortal(ulyssus).
  likes(X,iceCream).
  likes(mary,brotherOf(helen)).
- variables are implicitly universally quantified.

rules

- *rules* are conditional statements.
- e.g. mortal(X) :- human(X).
  i.e. $\forall x$ Human(x) $\rightarrow$ Mortal(x),
  all humans are mortal.
- daughter(X,Y) :- father(Y,X), female(X).
- , represents conjunction.
- likes(mary,X) :- isSweet(X).

rules

- ancestor(X,Y) :- father(X,Z), ancestor(Z,Y).
- variables are universally quantified from outside; can think of variables that appear only in rule body as existentially quantified.

queries

- A query asks whether a given statement is true, i.e. whether it follows from the program.
- e.g. ?- mortal(ulyssus). given
  mortal(X) :- human(X).
  human(ulyssus). human(penelope).
  god(zeus).
  Prolog answers Yes

queries

- ?- mortal(X).
  X = ulyssus ;
  X = penelope
  Yes
- variables in queries are existentially quantified; can be used to retrieve information.
- can have conjunctive queries, e.g.
  ?- mortal(X), mortal(Y), not(X = Y).

lists

- lists are a special kind of term that allows arbitrary number of components
- [] is the empty list
- .(a,b) is a dotted pair
- [a, b, c] = .(a,.(b,.(c,[]))) is a list of 3 components.
- the functor . builds binary trees
- can use display(X) to print internal representation of X

lists

- can refer to the first and rest of a list using the notation: [First | Rest]
- e.g. ?- X = [a,b,c], X = [F|R].
  X = [a,b,c]
  F = a
  R = [b,c]
- E.g. X = [b], Y = a, Z = [Y|X].
  X = [b]
  Y = a
  Z = [a,b]

unification

- this was an instance of the kind of pattern matching called unification that Prolog performs
- Prolog tries to find a way to instantiate the variables (substitute terms for them) that satisfies the query
- more on this later

terms can represent graphs

- ?- X = [a|X].
  X = [a, a, a, a, a, a, a, a, a|…]
  Yes
- here X denotes an infinite or circular list
- this is not allowed in first-order logic; a variable cannot denote a term and one of its subterms; but Prolog omits the "occurs check"

building a knowledge base

- to be used in a computation, facts and rules must be stored in the (dynamic) database
- facts and rules get into the database through *assertion* and *consultation*
- consultation loads facts and rules from a file

assertion

- ?- assert(human(ulyssus)).
- ?- human(X).
  X = ulyssus
  Yes
- assertion can be done dynamically
- also retract to remove facts and rules from the DB
- like assignment, change state; avoid when possible

consultation

- ?- consult('family.pl').
  loads facts and rules from file family.pl
- ?- [family].
  does the same thing
- ?- [user].
  lets you enter facts and rules from the keyboard

denotation/meaning of Prolog programs

- a Prolog program defines a set of relations, i.e. specifies which tuples of objects/terms belong to a particular relation
- in logic, this is called a model
- declarative programming is very different from usual procedural programming where programs perform many state changing operations

denotation of Prolog program e.g.

- ```
  fatherOf(john,paul).
  fatherOf(mary,paul).
  motherOf(john,lisa).
  parentOf(X,Y) :- fatherOf(X,Y).
  parentOf(X,Y) :- motherOf(X,Y).
  ```
- fatherOf is the relation {<john,paul>, <mary,paul>}
- what is the relation associated with motherOf and parentOf?
  ```

rules as procedures

- rule has form goal :- body
- goal or head is like name of procedure
- terms on the RHS are like the body of the procedure, the sub-goals that have to be achieved to show that the goal holds
- the sub-goals will be attempted left-to-right
- rule succeeds if all sub-goals succeed

passing values

- calling/querying a goal can instantiate its variables
- a sub-goal's success can bind a variable within it, also binding the same variable in the goal
- binding or instantiating a variable is giving it a value
- compare to passing values into or out of a procedure

passing values e.g.

- Assume program:
  motherOf(john,lisa).
  parentOf(X,Y) :- motherOf(X,Y).
- Queries:
  ?- parentOf(john,X).
  X = lisa Yes
  ?- parentOf(X,lisa).
  X = john Yes
  ?- parentOf(X,Y).
  X = john, Y = lisa Yes
- No fixed input and output parameters

almost everything is syntactically a term

- lists are terms; what is the functor?
- rules are terms:
  grandfather(X,Y):- father(X,Z), father(Z,Y).
  What are the functors?
- queries are terms

operators

- some functors are represented by *infix* or *prefix* or *postfix* operators
- Some infix operators: is, =, +, *, /, mod, >, >=, ":-", ",", etc.
- + and - are both prefix and infix
- :- as prefix is a command, used for declarations
- operators have precedence
- can define our own operators

arithmetic functions

- Prolog retains arithmetic functions as functions (more intuitive):

  ?- X **is** exp(1). % exp(1) = e^1

  X = 2.71828

  Yes

  ?- X is (4 + 2) * 5.

  X = 30

  Yes
- How does is compare with =, assignment?

relational thinking

- in Prolog, formulate statements about function values as relational facts, e.g.

  factorial(0,1).

  factorial(N,M):- K is N -1, factorial(K,L),
                   M is N * L.

- to compose functions, e.g. Y = f(g(X)), you must name intermediate results

  fg(X,Y):- g(X,Z), f(Z,Y).

help is sometimes helpful

?- help(reverse).

reverse(+List1, -List2)

Reverse the order of the elements in List1 and unify the result with the elements of List2.

+arg: arg is input and should be instantiated.

-arg: arg is output and can be initially uninstantiated; if the query succeeds, the arg is instantiated with the "output" of the query.

?arg: arg can be either input or output

online help

?- help(lists).
No help available for lists
Yes
?- apropos(lists).

merge/3	Merge two sorted lists
append/3	Concatenate lists
Section 11-1	"lists: List Manipulation"
Section 15-2-1	"lists”

Yes
?- help(append/3).
append(?List1, ?List2, ?List3)
Succeeds when List3 unifies with the concatenation of List1 and List2. The predicate can be used with any instantiation pattern (even three variables).

examples

```
?- append([a,b],[c],X).
X = [a, b, c]

Yes
?- append(X,[c],[a,b,c]).
X = [a, b]

Yes
?- append([a,b],[c],[a,b,d]).

No
```

more examples

```
?- append([a,b],X,Y).
X = _G187
Y = [a, b|_G187]
Yes
?- append(X,Y,Z).
X = []
Y = _G181
Z = _G181 ;

X = [_G262]
Y = _G181
Z = [_G262|_G181] ;

X = [_G262, _G268]
Y = _G181
Z = [_G262, _G268|_G181]
```

append is an example of a reversible or **steadfast** predicate (Richard O'Keefe)

reversible programming

- good predicates are steadfast
- they gives correct answers even if unusual values are supplied
  - e. g. variables for inputs, constants for outputs
- non-steadfast predicates require specific arguments to be instantiated (input) or variables (output)

unification

- Prolog matches terms by *unifying* them, i.e. applying a most general unifier to them
- it instantiates variables as little as possible to make them match, e.g.

```
?- X = f(Y,b,Z), X = f(a,V,W).
X = f(a, b, _G182)
Y = a
Z = _G182
V = b
W = _G182
```


family relations example

family relations

- the database:

  rules

```
parent(Parent, Child) :- mother(Parent, Child).
parent(Parent, Child) :- father(Parent, Child).
```

  facts

```
father('George', 'Elizabeth'). father('George', 'Margaret').
mother('Mary', 'Elizabeth').  mother('Mary', 'Margaret').
```

- Note encoding of disjunction

finding all solutions

```
| ?- parent(Parent, Child).
Parent = 'Mary',
Child = 'Elizabeth' ;

Parent = 'Mary',
Child = 'Margaret' ;

Parent = 'George',
Child = 'Elizabeth' ;

Parent = 'George',
Child = 'Margaret' ;

no
```

how prolog finds solutions

```
trace]  ?-
    parent(Parent, Child1),
    parent(Parent, Child2),
    not(Child1 = Child2).
   Call: (8) parent(_G313,
    _G314) ? creep
Call: (9) mother(_G313,
    _G314) ? creep
Exit: (9) mother('Mary',
    'Elizabeth') ? creep
Exit: (8) parent('Mary',
    'Elizabeth') ? creep
Call: (8) parent('Mary', _G317)
    ? creep
Call: (9) mother('Mary',
    _G317) ? creep
Exit: (9) mother('Mary',
    'Elizabeth') ? creep
Exit: (8) parent('Mary',
    'Elizabeth') ? creep
Redo: (9) mother('Mary',
    _G317) ? creep
Exit: (9) mother('Mary',
    'Margaret') ? creep
Exit: (8) parent('Mary',
    'Margaret') ? creep

Parent = 'Mary'
Child1 = 'Elizabeth'
Child2 = 'Margaret'
```


Prolog's query answering process

- a query is a conjunction of terms
- answer to the query is *yes* if all terms *succeed*
- A term in a query *succeeds* if
  - it matches a fact in the database *or*
  - it matches the head of a rule whose body succeeds
- the substitution used to unify the term and the fact/head is applied to the rest of the query
- works on query terms in **left to right order**; databases facts/rules that match are tried in **top to bottom** order

recursion examples

generating permutations

- A permutation P of a list L is a list whose first is some element E of L and whose rest is a permutation of L with E removed.
- [] is a permutation of []
- In Prolog:

```
permutation([],[]).
permutation(L,[E|PR]) :- select(E,L,R),
        permutation(R,PR).
```

selecting an element from a list

- To select an element from a list, can either select the first leaving the rest, or select some element from the rest and leaving the first plus the unselected elements from the rest.
- In Prolog:

```
select(X,[X|R],R).
select(X,[Y|R],[Y|RS]):- select(X,R,RS).
```


sorting by the definition

- Find a permutation that is ordered

```
sort(L,P):- permutation(L,P),
            ordered(P).
ordered([]).
ordered([E]).
ordered([E1,E2|R]) :- E1 <= E2,
            ordered([E2|R]).
```

- an example of “generate and test”

reverse

- reverse(L,RL) holds if RL is a list with the components of L reversed
- ordinary recursive definition

```
reverse([],[]).
reverse([F|R],RL):- reverse(R,RR),
    append(RR, [F], RL).
append([],L,L).
append([F|R],L,[F|RL]):-
    append(R,L,RL).
```


reverse

- Tail recursive definition:

```
reverse(L,RL):- reverse(L,[],RL).
reverse([],Acc,Acc).
reverse([F|R],Acc,RL):-
          reverse(R,[F|Acc],RL).
```

- recursive call is last thing done
- can avoid saving calls on stack

solving a logic puzzle with Prolog

the zebra puzzle

1. There are 5 houses, occupied by politically-incorrect gentlemen of 5 different nationalities, who all have different coloured houses, keep different pets, drink different drinks, and smoke different (now-extinct) brands of cigarettes.
2. The Englishman lives in a red house.
3. The Spaniard keeps a dog.
4. The owner of the green house drinks coffee.

...

6. The ivory house is just to the left of the green house.

...

11. The Chesterfields smoker lives next to a house with a fox.

Who owns the zebra and who drinks water?

Prolog implementation

- represent the 5 houses by a structure of 5 terms

  house(Colour, Nationality, Pet, Drink, Cigarettes)
- create a partial structure using variables, to be filled by the solution process
- specify constraints to instantiate variables

house building

```
makehouses(0,[]).

makehouses(N,[house(Col, Nat, Pet, Drk, Cig)|List])
        :- N>0, N1 is N - 1, makehouses(N1,List).
```

or more cleanly with anonymous variables:

```
makehouses(N,[house(_, _, _, _, _)|List])
        :- N>0, N1 is N - 1, makehouses(N1,List).
```

Why is this equivalent? (See p. 159.)

the empty houses

```
?- makehouses(5, List).

List = [house(_G233, _G234, _G235, _G236, _G237),
   house(_G245, _G246, _G247, _G248, _G249),
   house(_G257, _G258, _G259, _G260, _G261),
   house(_G269, _G270, _G271, _G272, _G273),
   house(_G281, _G282, _G283, _G284, _G285)]
```


constraints

- The Englishman lives in a red house.
  `house(red, englishman, _, _, _) on List,`
- The Spaniard keeps a dog.
  `house( _, spaniard, dog, _, _) on List,`
- The owner of the green house drinks coffee.
  `house(green, _, _, coffee, _) on List`
- The ivory house is just to the left of the green house
  ```
  sublist2( [house(ivory,      _,   _,    _,    _)
     ,house(green,      _,   _,    _,    _)], List),
  ```
- The Chesterfields smoker lives next to a house with a fox.
  ```
  nextto(house(   _,      _,   _,    _, chesterfields),
       house(   _,      _,  fox,    _,      _), List),
  ```

defining the on operator

- on is a user-defined infix operator that is a version of member/2
- :- op(100,xfy,on).

  X on List :- member(X,List).

  amounts to

  ```
  X on [X|_].
  X on [_|R]:- X on R.
  ```


predicates for defining constraints

- "just to the left of"? "lives next to"?
- define sublist(S,L)

  sublist2([S1, S2], [S1, S2 | _]) .

  sublist2(S, [_ | T]) :- sublist2(S, T).
- define nextto predicate

  nextto(H1, H2, L) :- sublist2([H1, H2], L).

  nextto(H1, H2 ,L) :- sublist2([H2, H1], L).

translating the constraints

- The ivory house is just to the left of the green house
  sublist2([house(ivory, _, _, _, _),
                  house(green, _, _, _, _)], List),
- The Chesterfields smoker lives next to a house with a fox.
  nextto(house(_, _, _, _, chesterfields),
            house(_, _, fox, _, _), List),

looking for the zebra

- Who owns the zebra and who drinks water?
  find(ZebraOwner, WaterDrinker) :-
        makehouses(5, List),
        house(red, englishman, _, _, _)   on List,
        … % all other constraints
        house(_, WaterDrinker, _, water, _) on List,
        house(_, ZebraOwner, zebra, _, _) on List.
- solution is generated and queried in the same clause
- neither water or zebra are mentioned in the constraints

solving the puzzle

```
?- [zebra].
% zebra compiled 0.00 sec, 5,360 bytes

Yes
?- find(ZebraOwner, WaterDrinker).

ZebraOwner = japanese
WaterDrinker = norwegian ;

No
```


how Prolog finds solution

After first 8 constraints:

```
List = [
house(red, englishman, snail, _G251, old_gold),
house(green, spaniard, dog, coffee, _G264),
house(ivory, ukrainian, _G274, tea, _G276),
house(green, _G285, _G286, _G287, _G288),
house(yellow, _G297, _G298, _G299, kools)]
```

how Prolog solves the puzzle

Then need to satisfy "the owner of the third house drinks milk", i.e.

List = [_, _, house(_, _, _, milk, _),_, _],

Can't be done with current instantiation of List. So Prolog will **backtrack** and find another.

how Prolog solves the puzzle

The unique complete solution is

L = [
house(yellow, norwegian, fox, water, kools),
house(blue, ukrainian, horse, tea, chesterfields),
house(red, englishman, snail, milk, old_gold),
house(ivory, spaniard, dog, orange, lucky_strike),
house(green, japanese, zebra, coffee, parliaments)]

See course web page for code of the example.