ENG2200
Electric Circuits

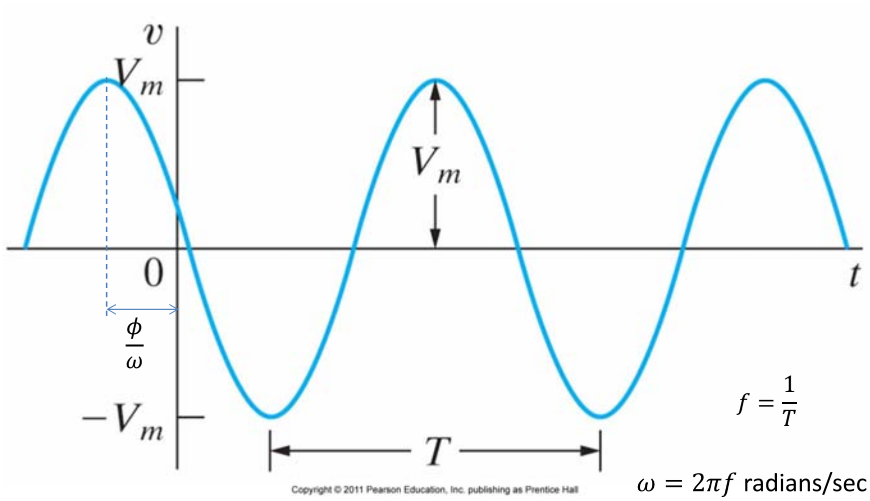
Chapter 9

Sinusoidal Steady State Analysis

Objectives

- Understanding phasor concept and be able to perform phasor transform and inverse phasor transform.
- Be able to transform a circuit with sinusoidal source into the frequency domain using phasor transform
- Know how to use circuits analysis techniques to solve circuits in the frequency domain.
- Be able to use phasor in analyzing circuits with ideal transformers.

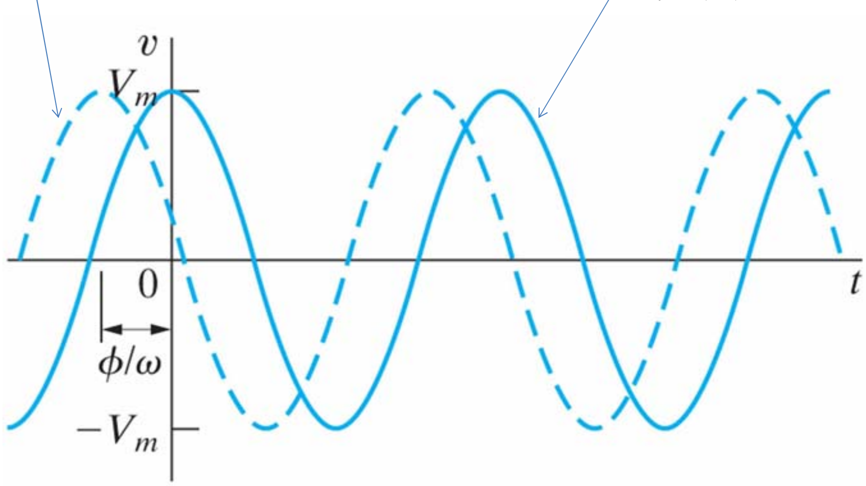
$v = V_m \cos(\omega t + \phi)$

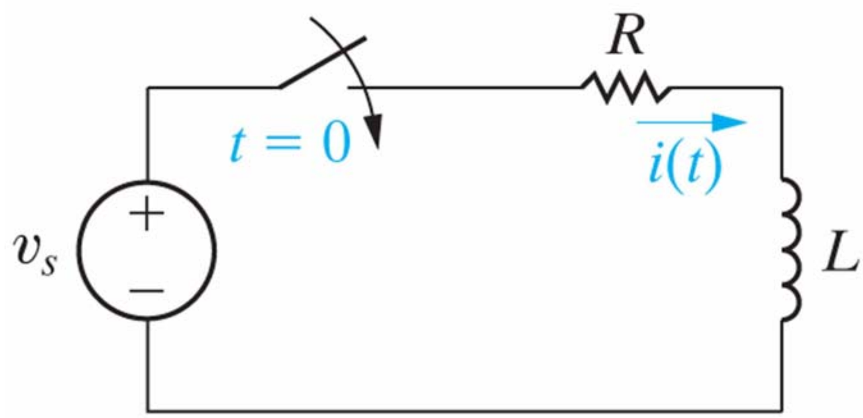


$f = \frac{1}{T}$

$\omega = 2\pi f$ radians/sec

$v = V_m \cos(\omega t + \phi)$ Phase shift of ϕ/ω $v = V_m \cos(\omega t)$





$$L\frac{di}{dt} + iR = V_m \cos(\omega t + \phi)$$

$$i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

RMS value

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Why RMS?

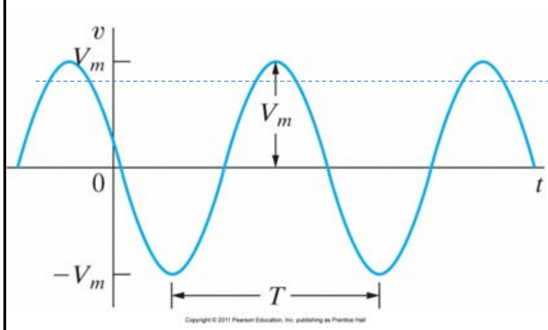
The Phasor

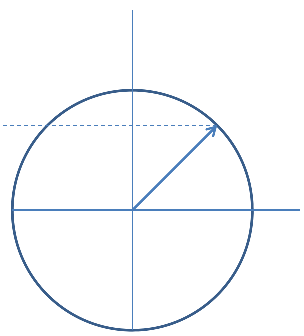
- The phasor is a complex number that carries the amplitude and phase angle information of a sinusoidal function.
- Euler's identity $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

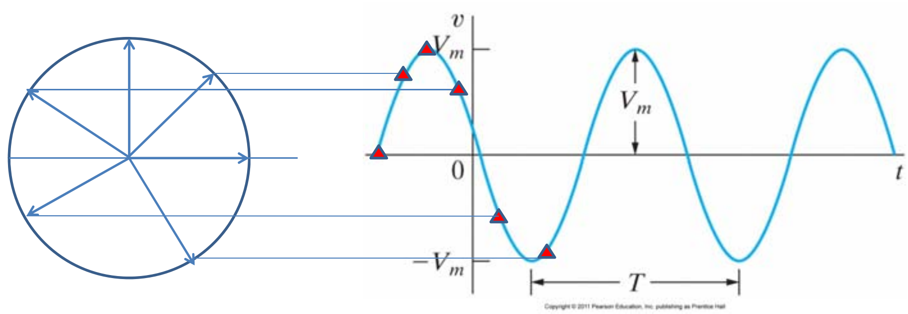
$$\cos\theta = \Re\{e^{j\theta}\}$$
$$\sin\theta = \Im\{e^{j\theta}\}$$

$$v = V_m \cos(\omega t + \phi)$$
$$v = \Re\{V_m e^{j\phi} e^{j\omega t}\}$$

$$Ae^{j\phi} = A\angle\phi^\circ$$







The inductor

$$v = L\frac{di}{dt}$$

$$v = V_m \cos(\omega t)$$

$$di = \frac{1}{L} V_m \cos(\omega t) dt$$

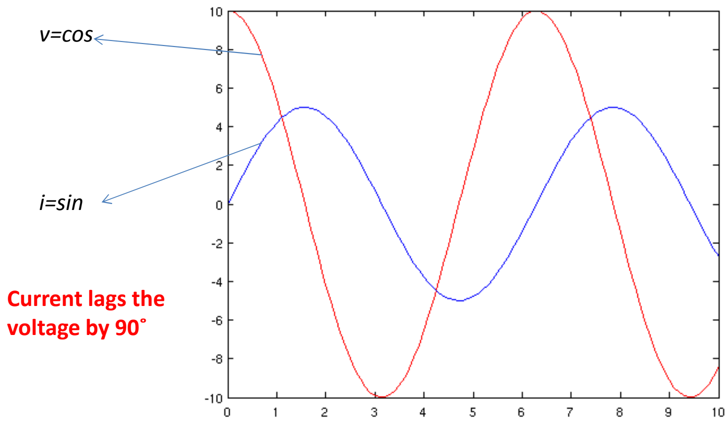
$$i = \frac{1}{L} V_m \int \cos(\omega t) dt$$

$$i = \frac{V_m}{\omega L}\sin(\omega t) = \frac{V_m}{\omega L}\cos(\omega t - \frac{\pi}{2})$$

$$I = \frac{V_m}{\omega L} e^{-j\pi/2} = \frac{V_m}{\omega L}\angle -\pi/2$$

$$Z = \frac{v}{i} = \omega L \angle \pi/2 = jwL$$

A plot showing the phase relationship between the current and voltage at the terminals of an inductor



The Capacitor

$$i = C\frac{dv}{dt}$$

$$v = V_m \cos(\omega t)$$

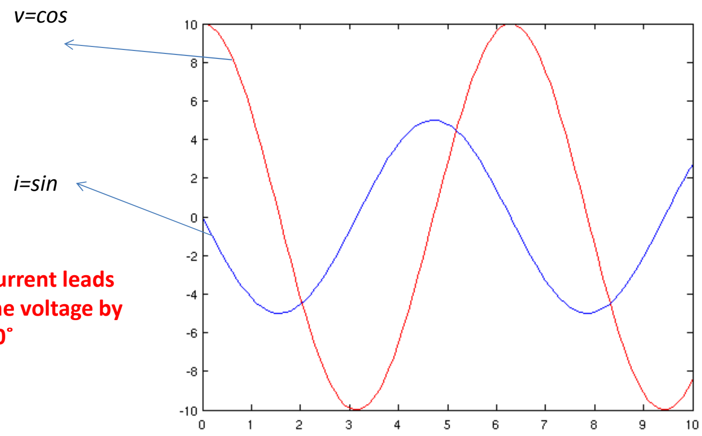
$$i = CV_m \frac{d}{dt}\cos(\omega t)$$

$$i = -C\omega V_m \sin(\omega t) = \omega C V_m \cos(\omega t + \frac{\pi}{2})$$

$$I = \omega C V_m e^{j\pi/2} = \omega C V_m \angle \pi/2$$

$$Z = \frac{v}{i} = \frac{1}{\omega C}\angle -\pi/2 = \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

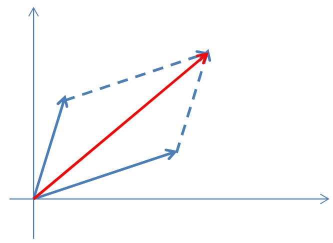
A plot showing the phase relationship between the current and voltage at the terminals of a capacitor



Current leads the voltage by 90˚

Adding Complex Numbers

$$
\begin{array}{rl}
 & x_1 + jy_1 \\
\pm & x_2 + jy_2 \\
\hline
 & (x_1 + x_2) \pm j(y_1 + y_2)
\end{array}
$$

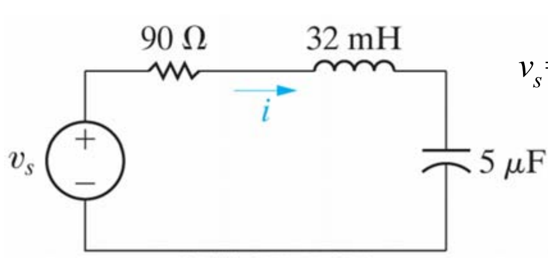


Multiplication

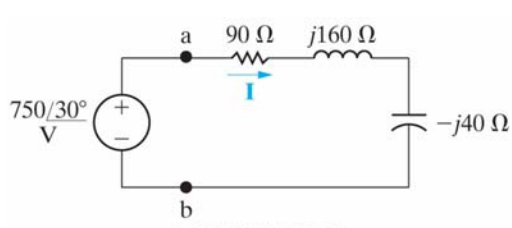
$$\begin{array}{r} x_1+jy_1 \\ \times \quad x_2+jy_2 \\ \text{-------------} \\ (x_1\,x_2-y_1y_2)+j(x_1y_2+x_2+y_1) \end{array}$$

$A_1\angle\theta_1 \times A_2\angle\theta_2 = A_1\,A_2\,\angle\theta_1+\theta_1$

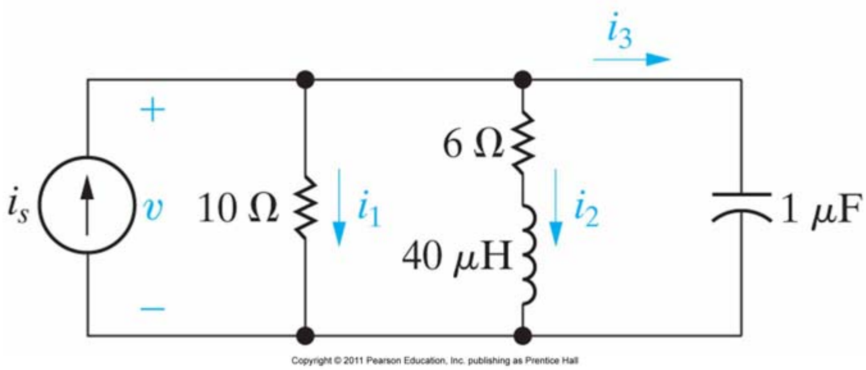
Example



v_s=700cos(5000t+30°)



Example



i_s=8cos200,000t

Example

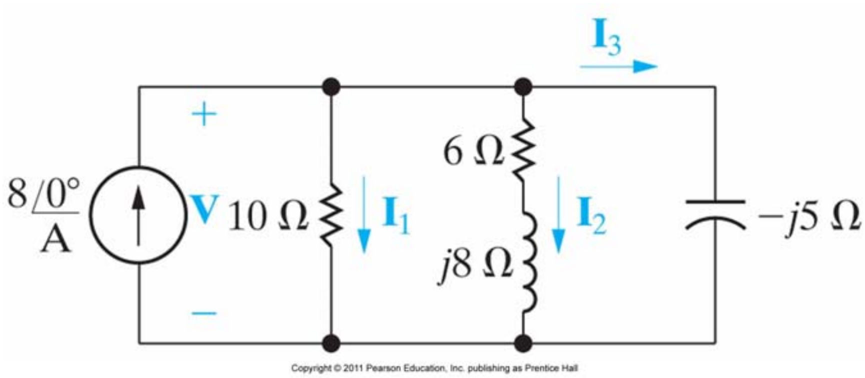


Figure 9.24 A source transformation in the frequency domain.

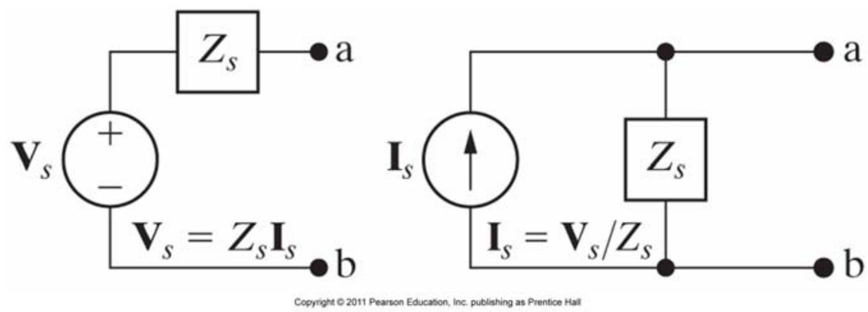


Figure 9.27 The circuit for Example 9.9.

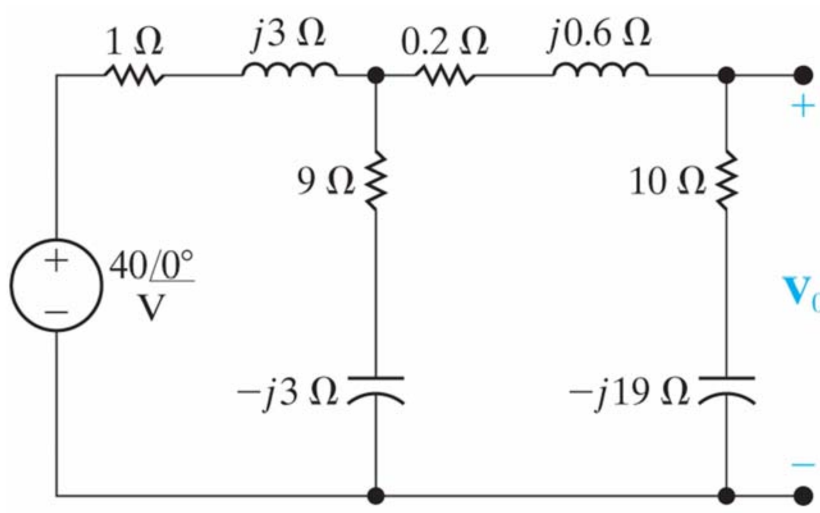


Figure 9.28 The first step in reducing the circuit shown in Fig. 9.27.

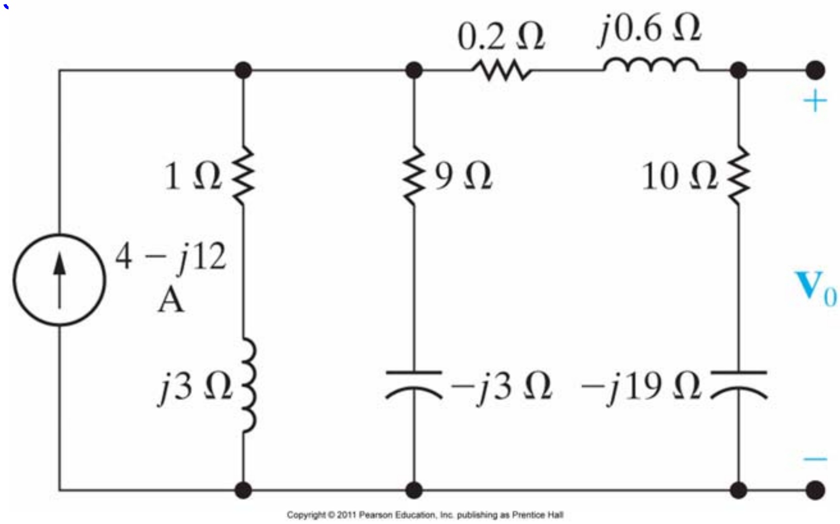


Figure 9.29 The second step in reducing the circuit shown in Fig. 9.27.

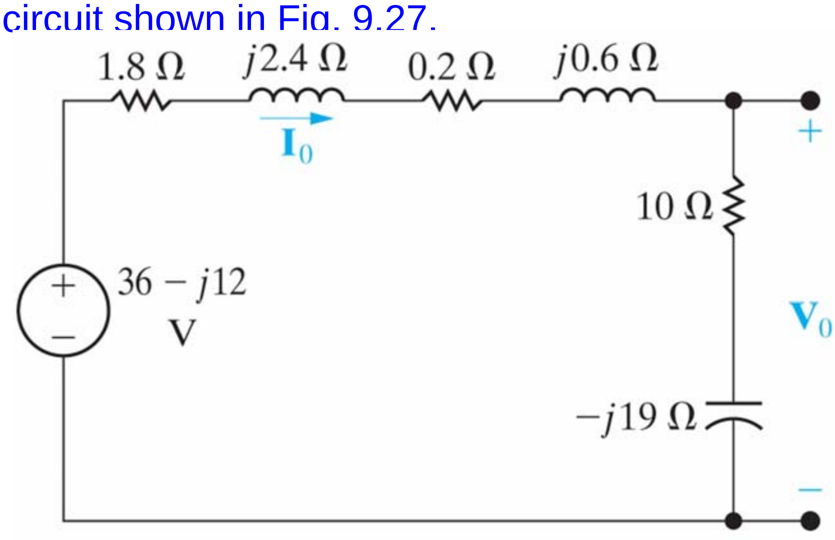


Figure 9.30 The circuit for Example 9.10.

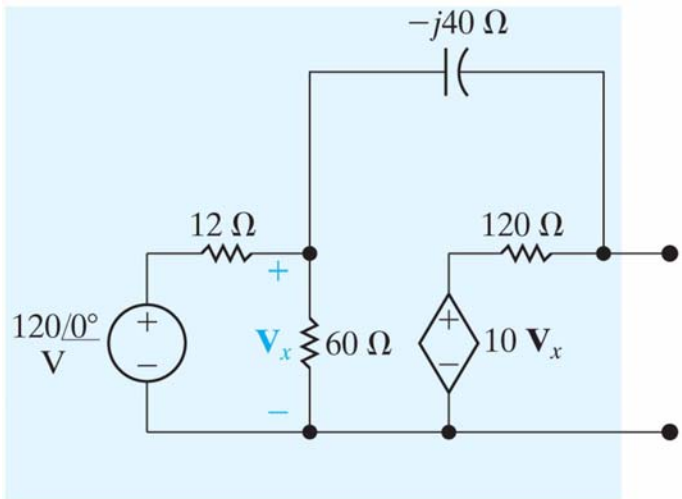


Figure 9.31 A simplified version of the circuit shown in Fig. 9.30.

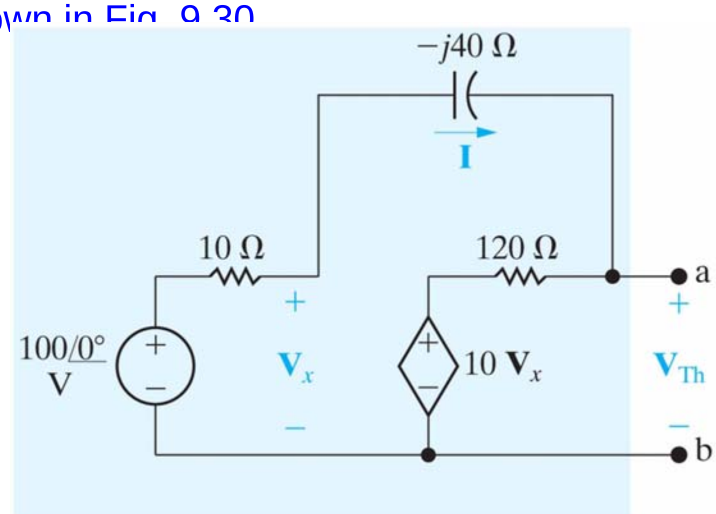


Figure 9.32 A circuit for calculating the Thévenin equivalent impedance.

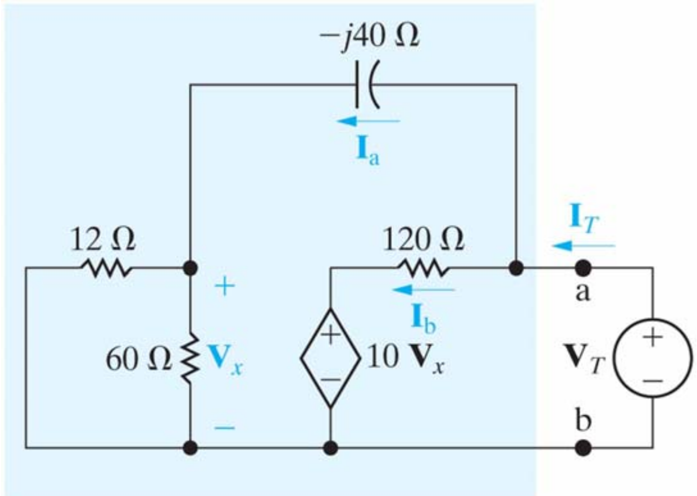


Figure 9.33 The Thévenin equivalent for the circuit shown in Fig. 9.30.

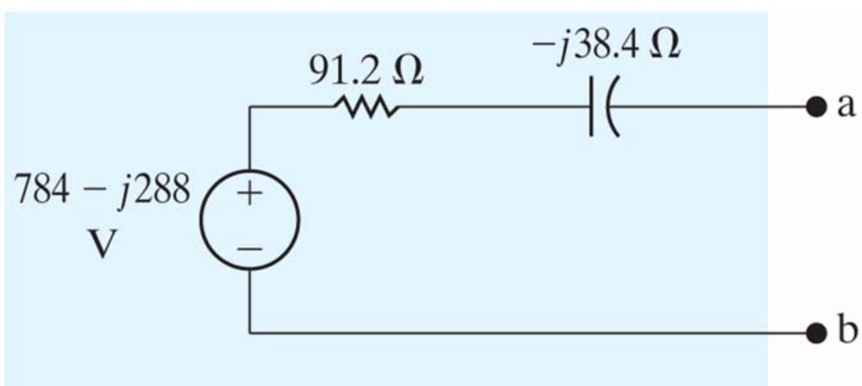


Figure 9.34 The circuit for Example 9.11.

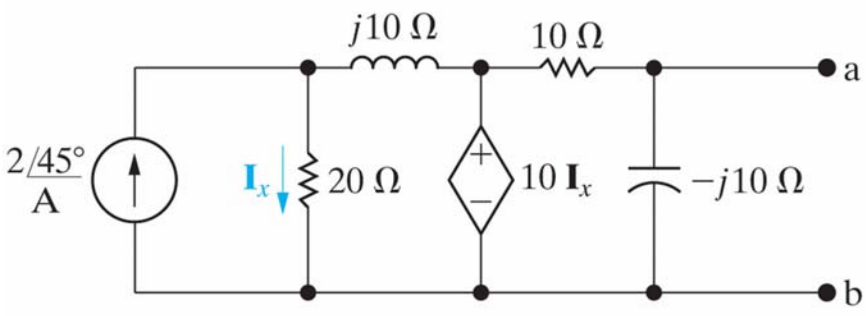


Figure 9.35 The circuit shown in Fig. 9.34, with the node voltages defined.

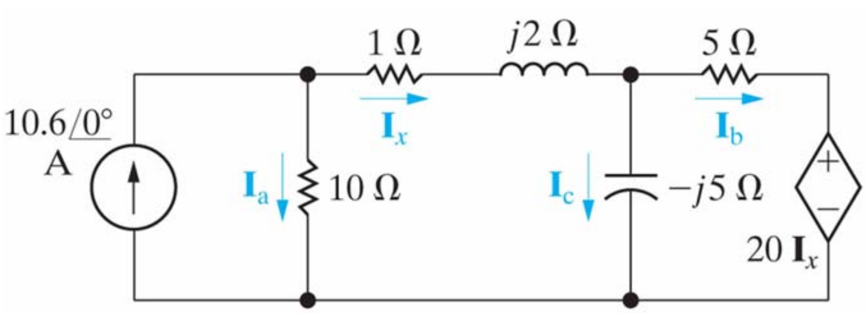


Figure 9.36 The circuit for Example 9.12.

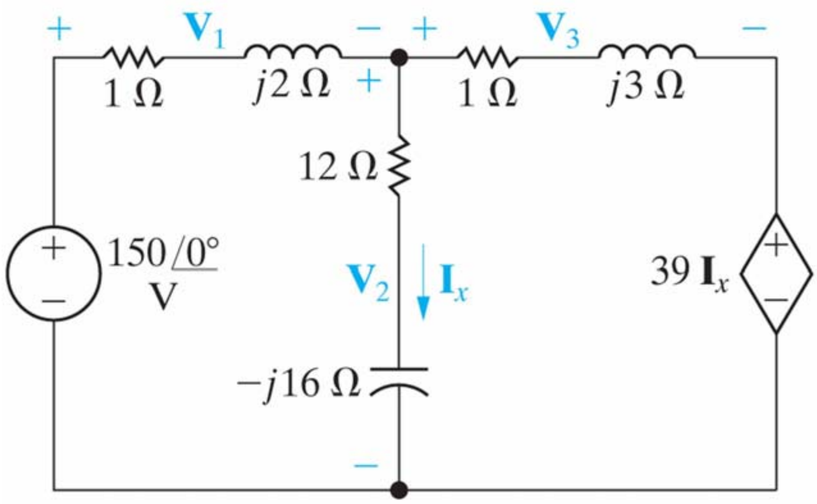
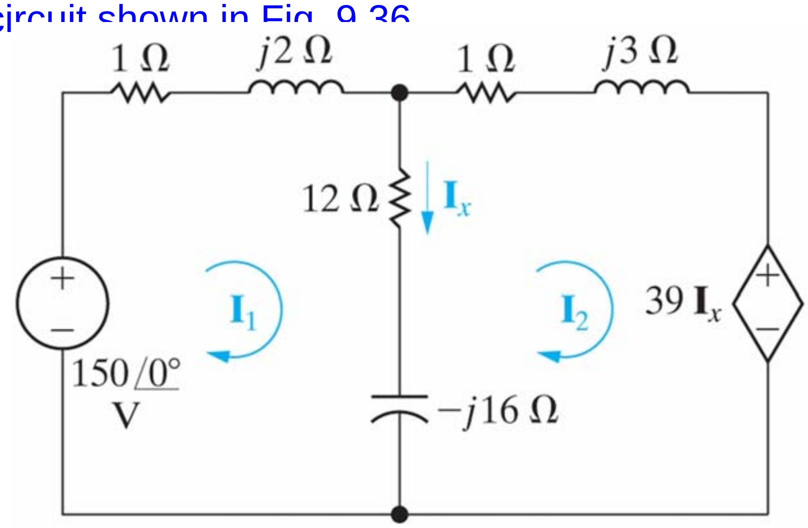
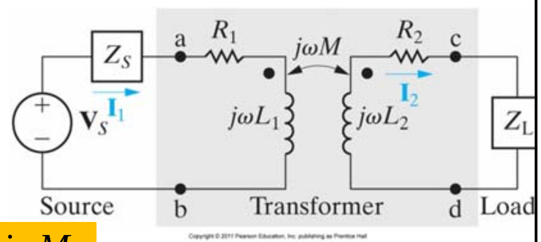


Figure 9.37 Mesh currents used to solve the circuit shown in Fig. 9.36





$$V_s = I_1(Z_s + R_1 + j\omega L_1) - I_2 j\omega M$$
$$0 = -j\omega M I_1 + I_2(R_2 + j\omega L_2 + Z_L)$$

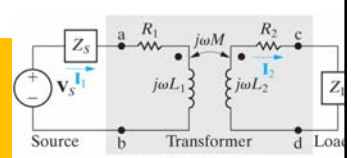
$$j\omega M I_1 = I_2(R_2 + j\omega L_2 + Z_L)$$
$$I_2 = \frac{j\omega M I_1}{(R_2 + j\omega L_2 + Z_L)}$$
$$Z_{11} = Z_s + R_1 + j\omega L_1$$
$$Z_{22} = Z_L + R_2 + j\omega L_2$$

$$I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_s$$
$$\frac{V_2}{I_1} = Z_{total} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$

Impedance in the primary loop

Reflected impedance

$$Z_{reflected} = \frac{\omega^2 M^2}{Z_{22}}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + R_L + j\omega X_L}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{(R_2 + R_L) + j\omega(L_2 + X_L)}$$
$$Z_{reflected} = \frac{\omega^2 M^2}{|Z_{22}|^2}\left[(R_2 + R_L) - j\omega(L_2 + X_L)\right]$$



The complex conjugate of the self impedance of the secondary circuit scaled by a factor