ENG2200
Electric Circuits

Chapter 4

Techniques for Circuit Analysis

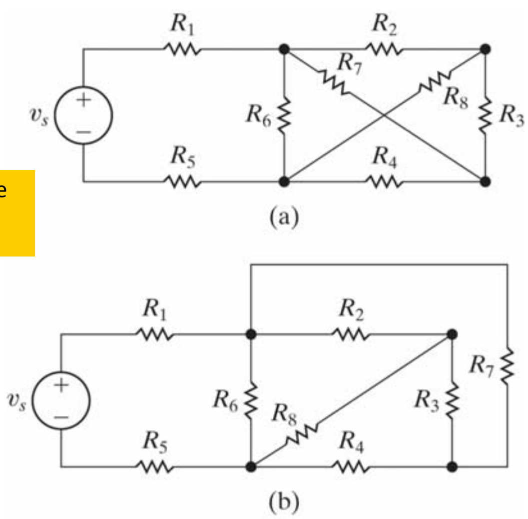
Objectives

- Understand and being able to use node-voltage and mesh-current methods to solve a circuit.
- Understand source transformation and being able to use it to solve a circuit.
- Understand the concept of Thevenin and Norton equivalent circuits and being able to construct them for any circuit.
- Know the condition for maximum power transfer to a resistive load, and being able to calculate the value of the load resistor that achieves that.

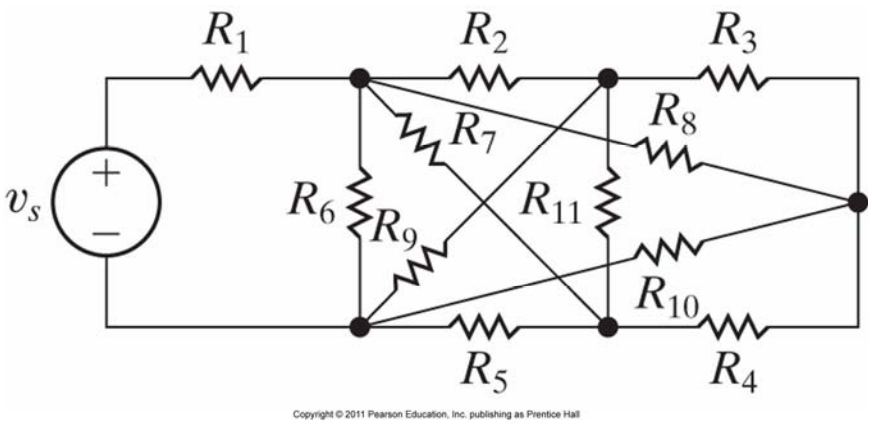
Terminology

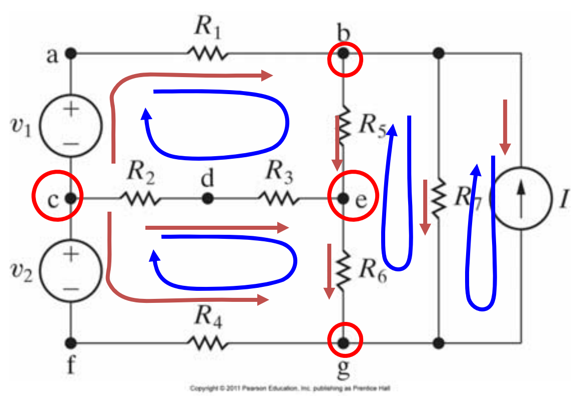
- **Node**: A point that connects two or more circuit elements.
- **Essential node**: A node where 3 or more circuit elements join.
- **Path**: A trace of adjoining basic elements with no element included twice.
- **Branch**: A path that connects two nodes.
- **Essential path**: A path which connects two essential nodes without passing through an essential node.
- **Loop**: A path whose last node is the same as the first
- **Mesh**: A loop that does not enclose any other loops
- **Planar circuit**: A circuit that could be drawn on a plane with no crossing branches.

Planar Circuit (same circuit).



Non Planar Circuit





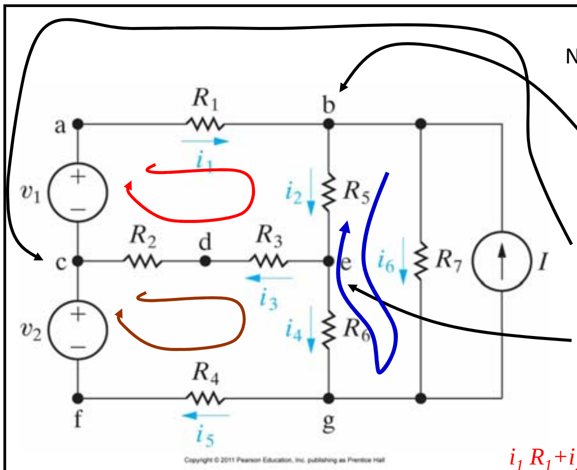
Essential nodes

Essential branches

Meshes

Simultaneous Equations

- In a circuit with *b* essential branches, and *n* essential nodes
- Get *n-1* equations by applying KCL at *n-1* nodes (why not n?)
- Still need *b-n+1* equations to solve for the currents in each essential branch.
- Use KVL on loops or meshes to get the remaining *b-n+1* equations
- Solve them.



Need 6 equations

$-i_1 + i_2 + i_6 - I = 0$

$i_1 - i_5 - i_3 = 0$

$i_4 - i_2 + i_3 = 0$

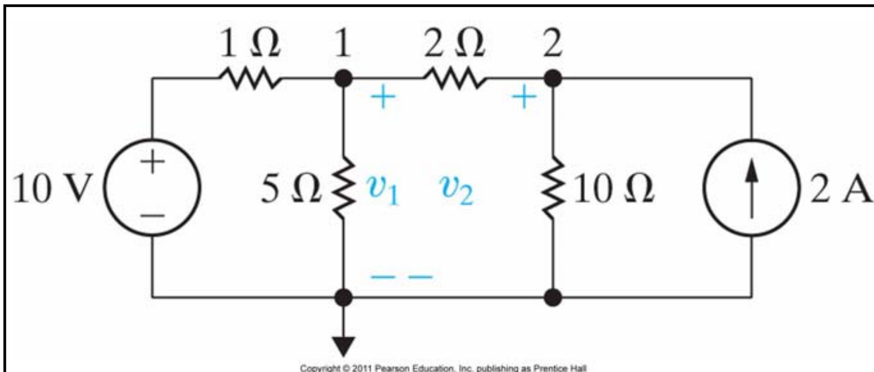
$i_1 R_1 + i_2 R_2 + i_3(R_3 + R_4) - v_1 = 0$

$-i_3 (R_2 + R_3) + i_4 R_6 + i_5 R_4 - v_2 = 0$

$i_2 R_5 + i_4 R_6 - i_6 R_7 = 0$

Node-Voltage Method

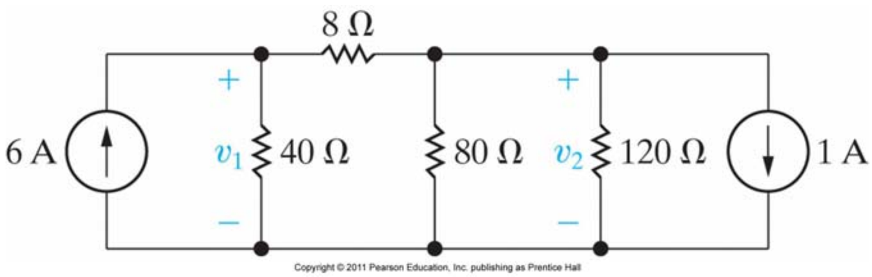
- Draw the circuit in a neat way
- If the circuit has n essential nodes, we need n-1 equations.
- Select one of the n nodes as a reference node (usually the one with the most branches).
- Define the node voltages for the n-1 nodes (voltage rise from reference to the node).
- Write KCL for the n-1 nodes as a function of the node voltage.



$$\frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0$$

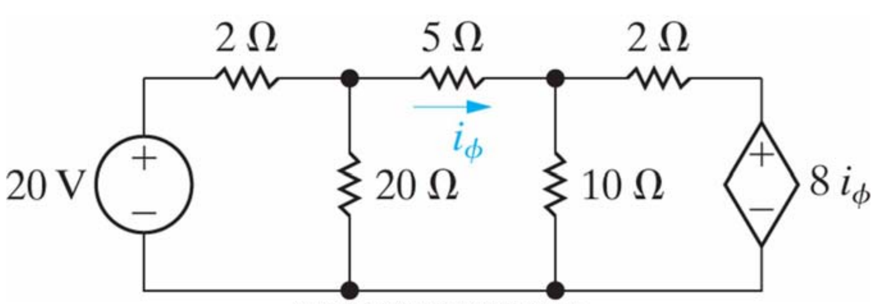
$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0$$

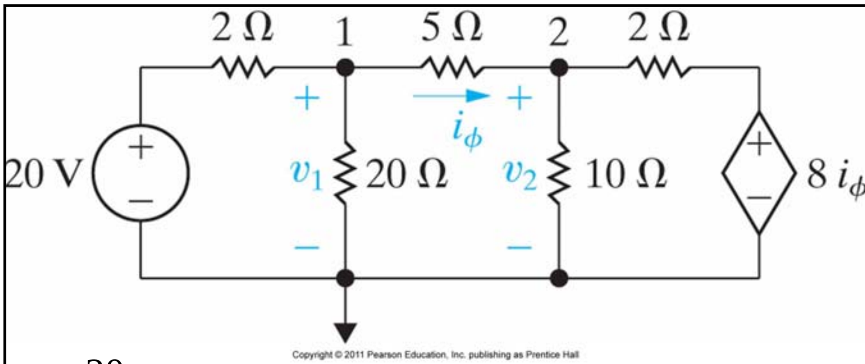
Example



Node-Voltage and Dependent Sources.

- Another constraint is added by the dependent source



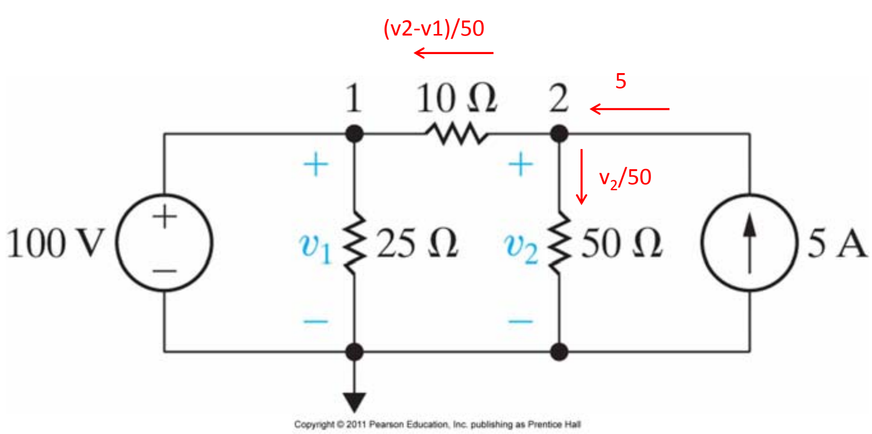


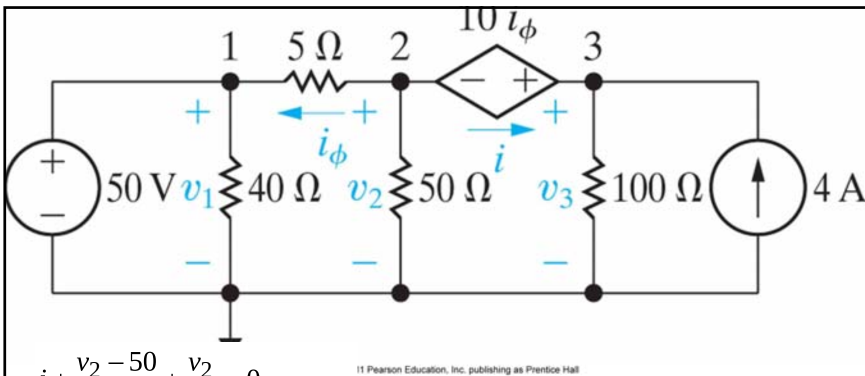
$$\frac{v_2 - 20}{2} + \frac{v_1 - v_2}{5} + \frac{v_1}{20} = 0$$

$$\frac{v_2 - 8i_\phi}{2} + \frac{v_2 - v_1}{5} + \frac{v_2}{10} = 0$$

$$i_\phi = \frac{v_1 - v_2}{5}$$

Special cases



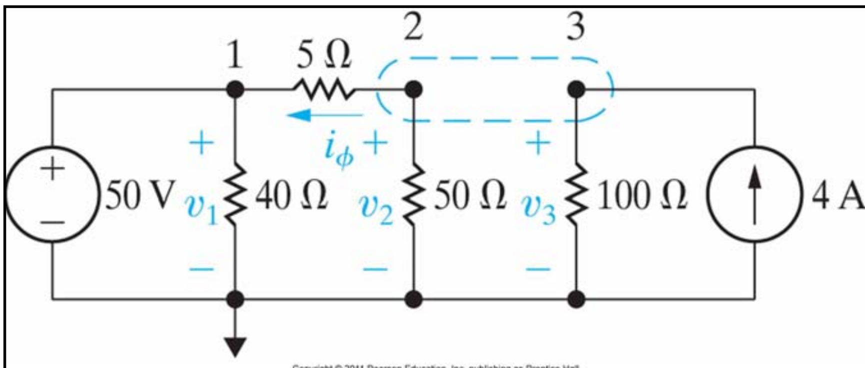


$$i + \frac{v_2 - 50}{5} + \frac{v_2}{50} = 0$$

$$\frac{v_3}{100} - 4 - i = 0$$

$$\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

Add these 2 equations. To get rid of i

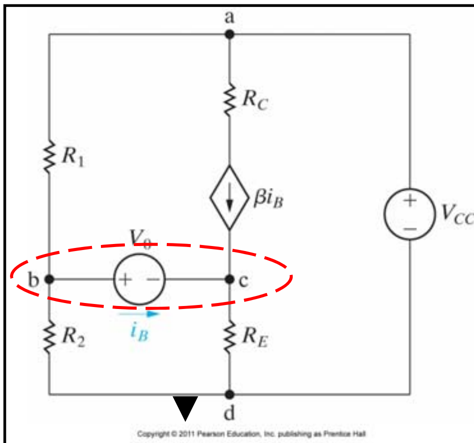


When we have a voltage source between 2 nodes, we can use the concept of a supernode to derive the previous equation

$$\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

Need one more equation in v_2 and v_3 to be able to solve the circuit

$v3=10i_\phi+v2$



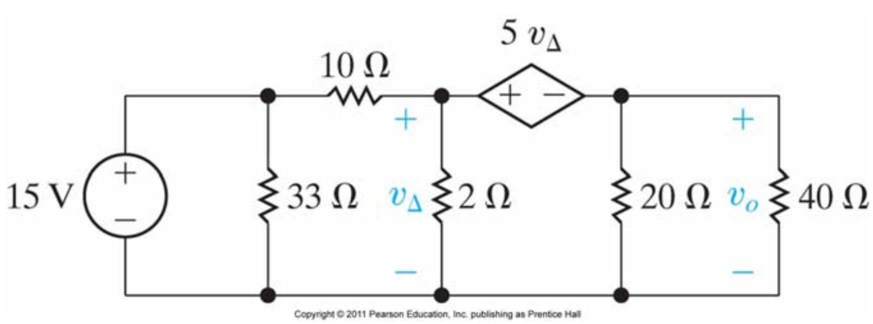
Using the concept of a super node

$$\frac{v_b - V_{CC}}{R_1} + \frac{v_b}{R_2} + \frac{v_c}{R_E} - \beta i_B = 0$$

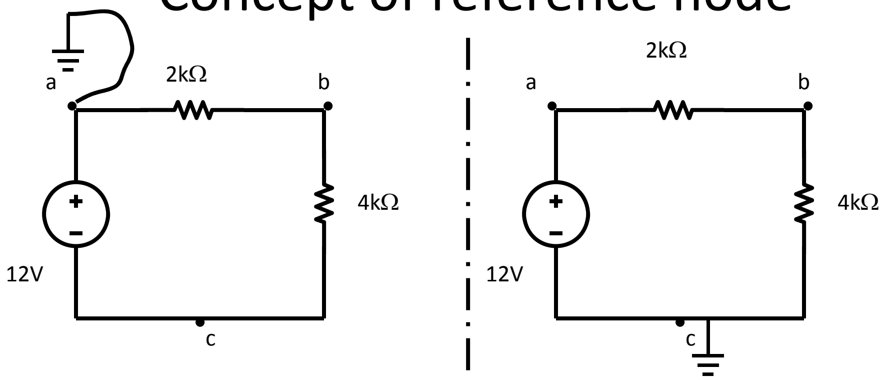
Need 2 more equation to get rid of v_c and i_B

$$v_c = v_b - V_0$$
$$vc = (i_B + \beta i_B)R_E$$

Example



Concept of reference node



2 different reference nodes

What is the voltages at a, b, c are they the same?

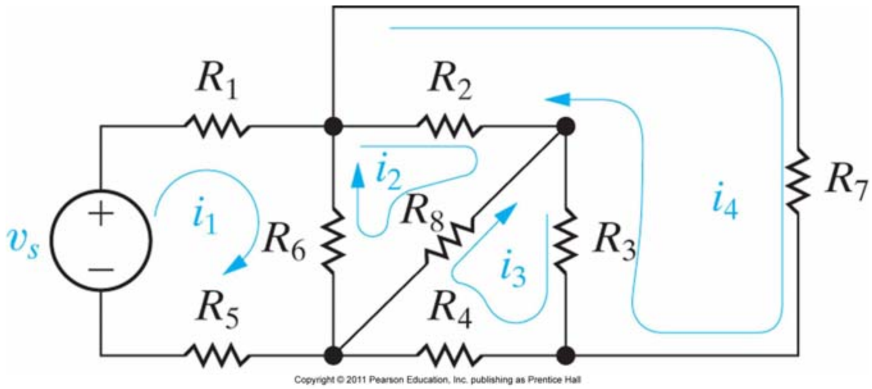
What about voltage differences?

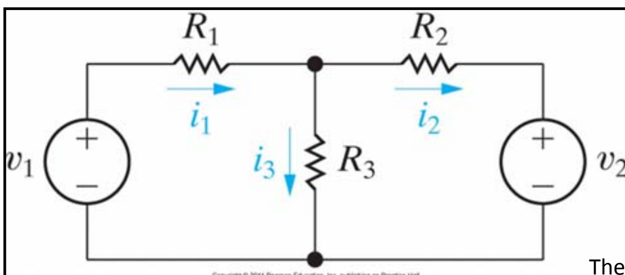
Mesh-Current Method

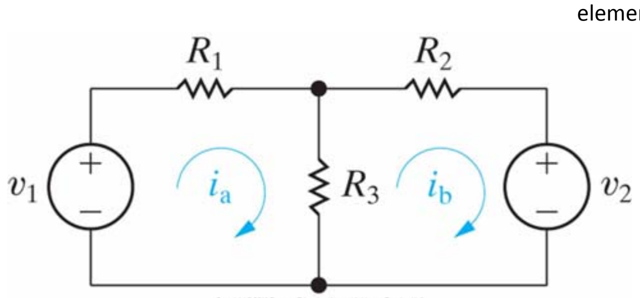
- For planar circuit
- A mesh current is a current that exists only in the perimeter in the mesh
- Mesh current may or may not be an actual current in a circuit element (i_1 and i_2)

7 branches vs. 4 meshes (the number of equations you need to solve the circuit

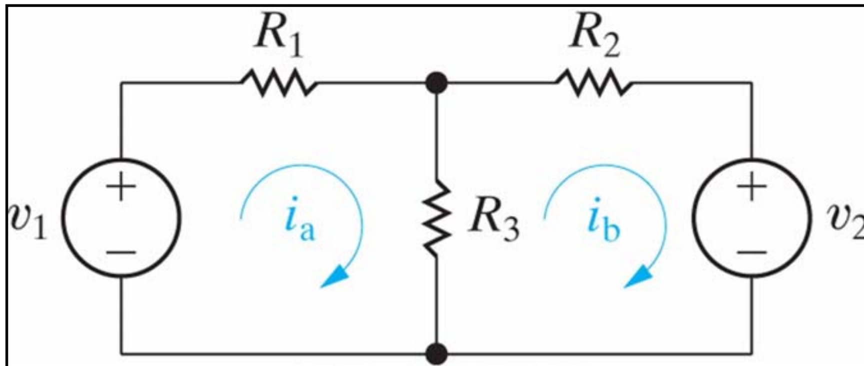
Is i_1 an actual current in a circuit element? What about i_2?





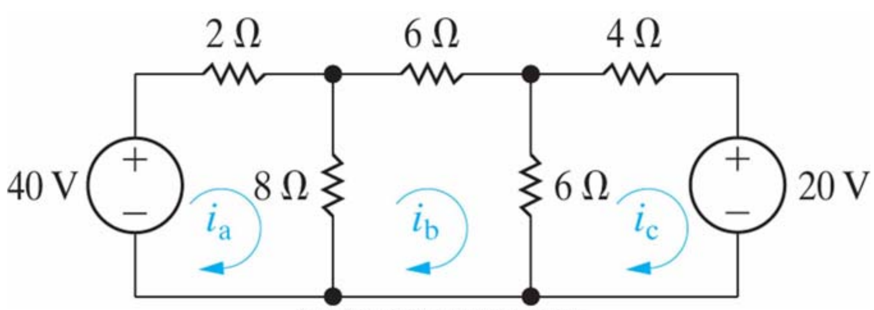


The relation between mesh currents and currents in a circuit element.



$$i_a R_1 + R_3(i_a - i_b) = v_1$$

$$R_3(i_b - i_a) + i_b R_2 = -v_2$$

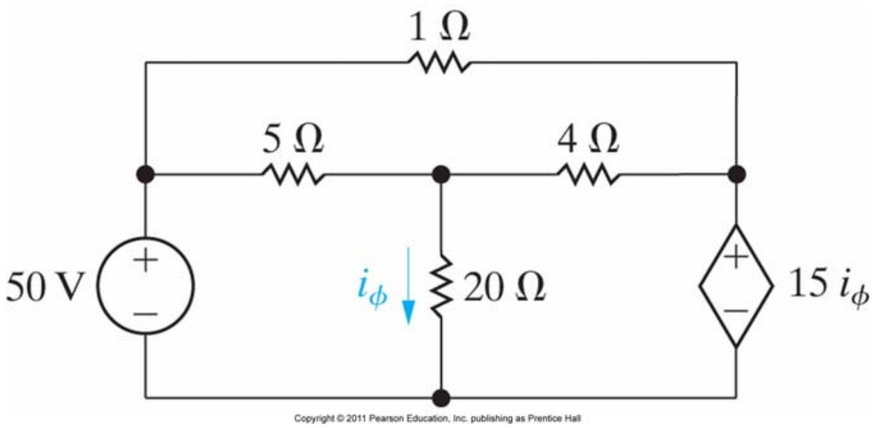


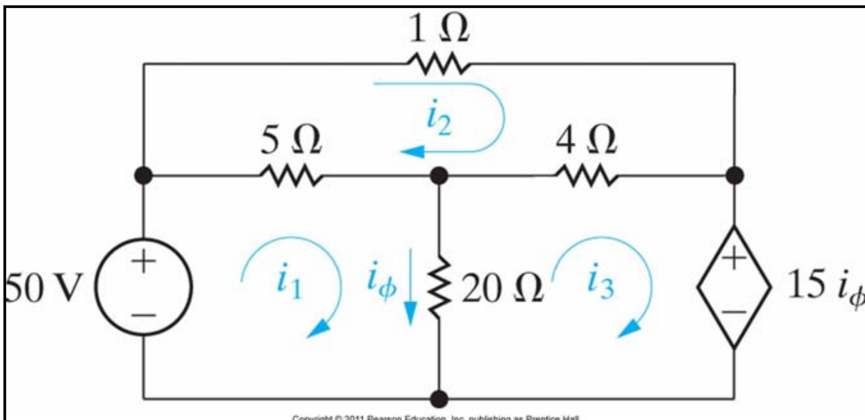
$$2i_a + 8(i_a - i_b) - 40 = 0$$

$$6i_b + 6(i_b - i_c) + 8(i_b - i_a) = 0$$

$$4i_c + 20 + 6(i_c - i_b) = 0$$

Mesh-Current with Dependent Sources

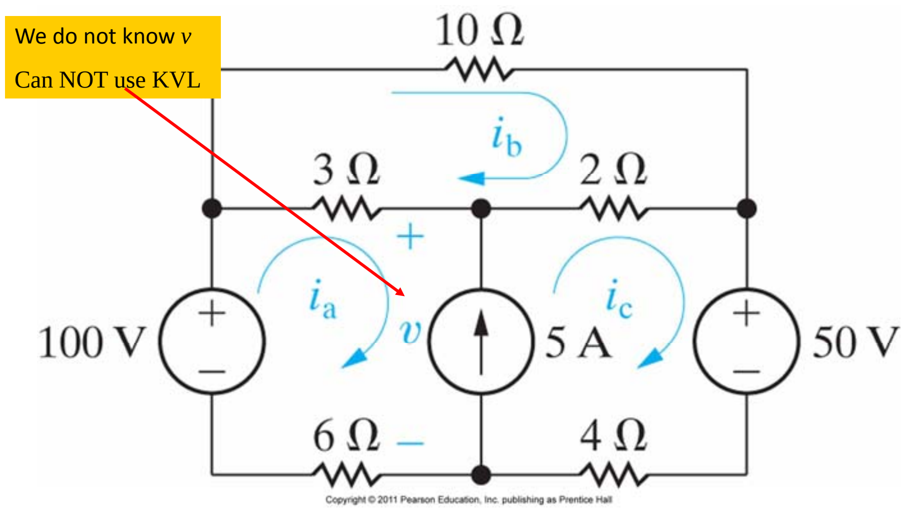




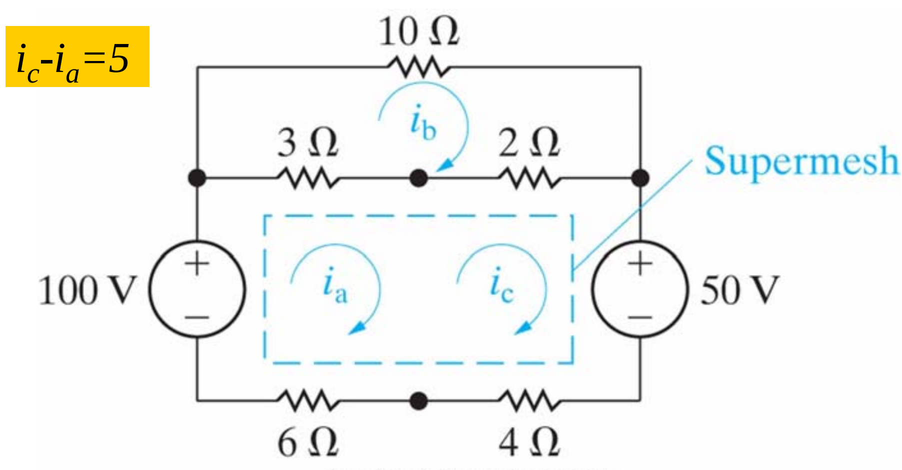
You can easily write 3 mesh-current equations. But what about i_ϕ

$i_\phi = i_1 - i_2$

Current Source between two nodes

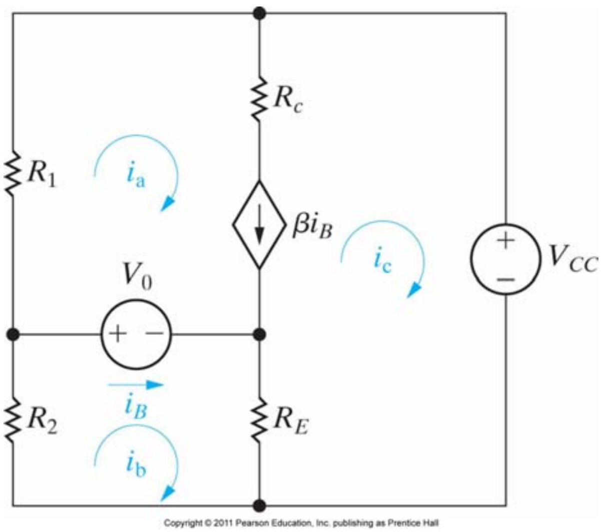


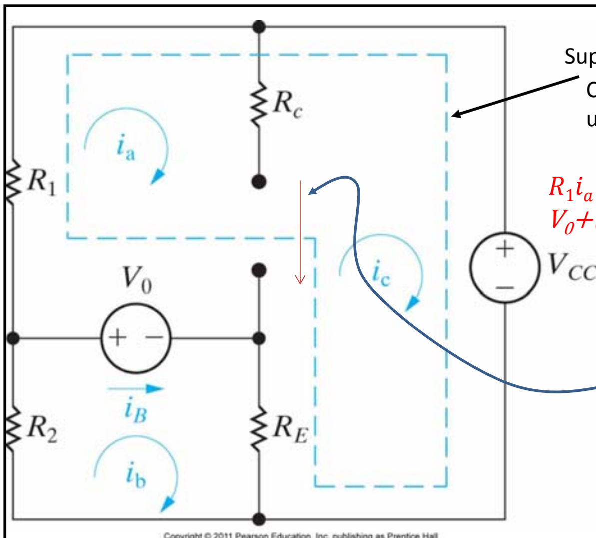
2 equations in 3 unknown



We do not know the voltage drop across the current source.

Can not write KVL for mesh a and c without introducing another variable





Supermesh

Can write 2 equations in 3 unknown, need one more

$R_1 i_a + vcc + Re(ic - ib) - V0 = 0$

$V_0 + (i_b - i_c)R_E + i_b R_2 = 0$

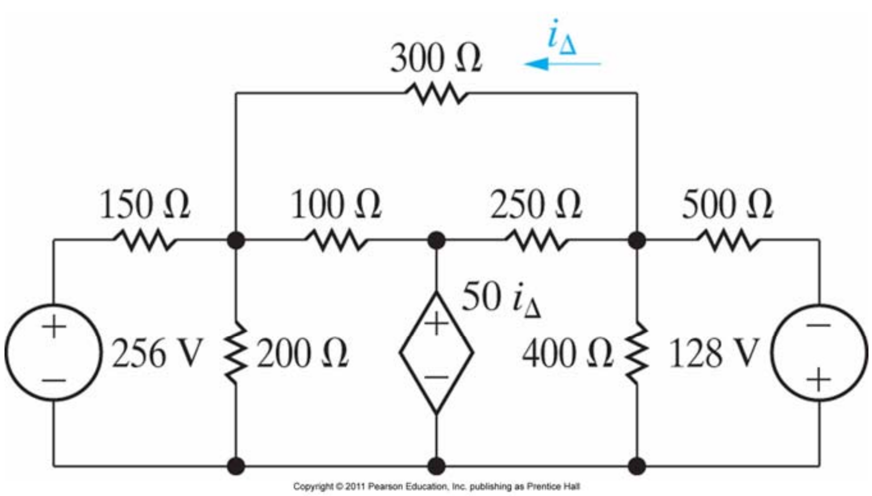
$$\beta i_B = i_a - i_c$$

$$i_B = i_b - i_c$$

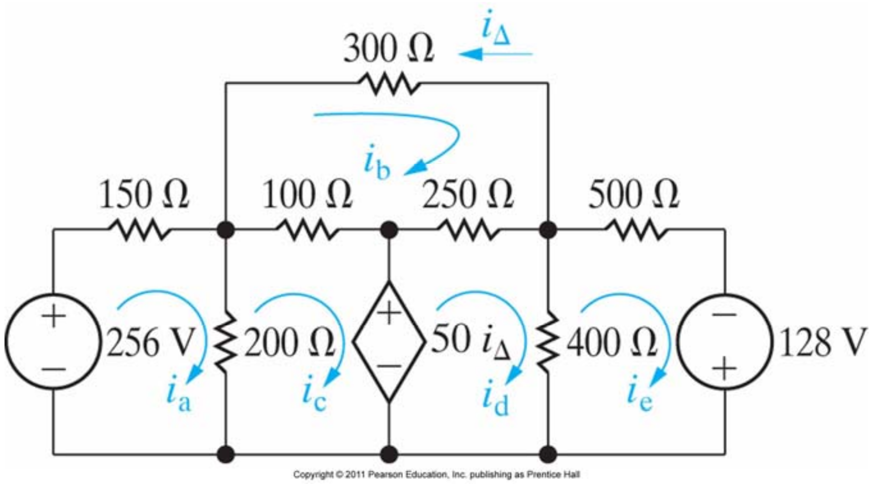
Node-Voltage vs. Mesh Current

- Which is better (less equations)
- Depnds
  - Number of equations formed for each
  - Any supernode (node voltage)
  - Supermesh? (mesh-current)
  - Do we have to solve for the entire circuit?

Find power in the 300 ohm R, which is better

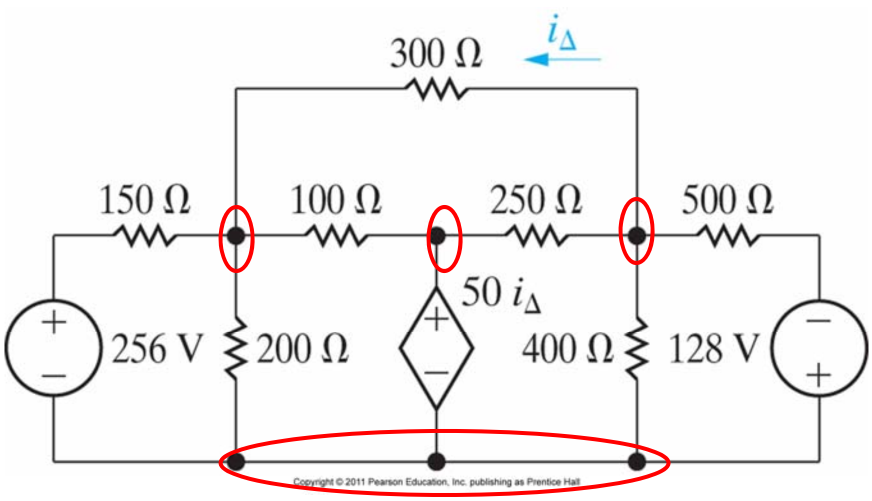


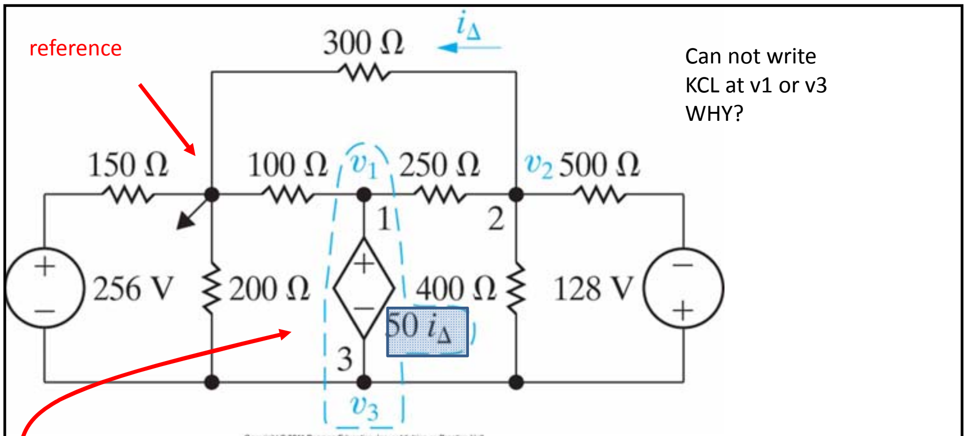
Mesh current: 5 equations (note that i_{Δ}=-i_b)



Node voltages: 4 nodes, need only 3 equations

Which node to choose a a reference node?

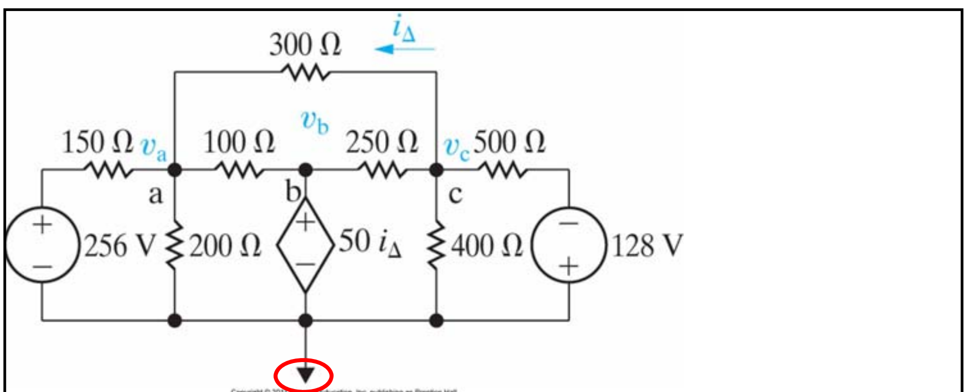




$$\frac{v_1}{100}+\frac{v_3}{200}+\frac{v_3+256}{150}+\frac{v_1-v_2}{250}+\frac{v_3-v_2}{400}+\frac{v_3-128-v_2}{500}=0$$

$$\frac{v_2-v_1}{250}+\frac{v_2+128-v_3}{500}+\frac{v_2-v_3}{400}+\frac{v_2}{300}=0$$ At node 2

$$v_1-v_3=50i_\Delta=50\frac{v_2}{300}$$ Constraint at supernode



$$\frac{v_a}{200}+\frac{v_a-256}{150}+\frac{v_a-v_b}{100}+\frac{v_a-v_c}{300}=0$$ Node a

$$\frac{v_c-v_b}{250}+\frac{v_c}{400}+\frac{v_c+128}{500}+\frac{v_c-v_a}{300}=0$$ Node c

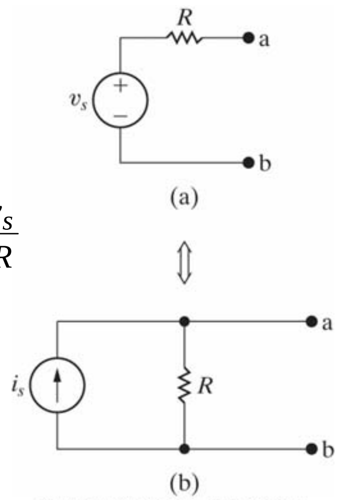
$$v_b=50i_\Delta=50\frac{v_c-v_b}{300}$$ Constraint at supernode

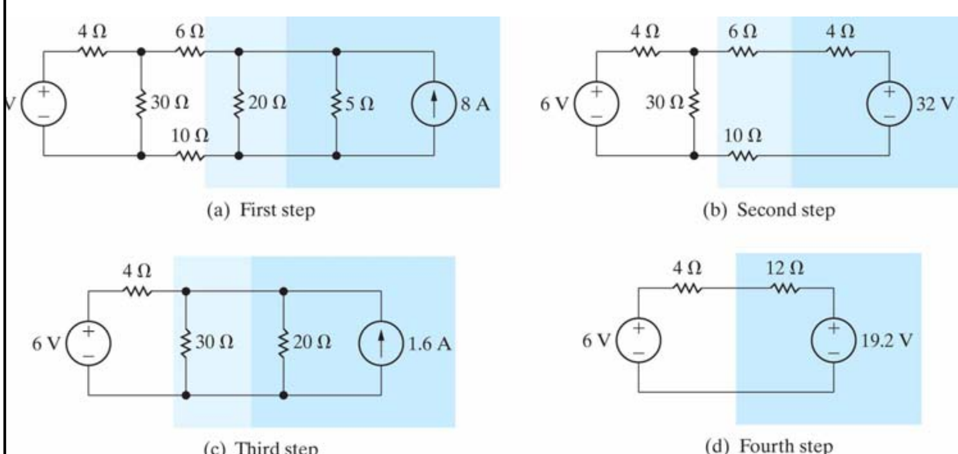
Source Transformation

- Some times it helps in reducing the circuit complexity

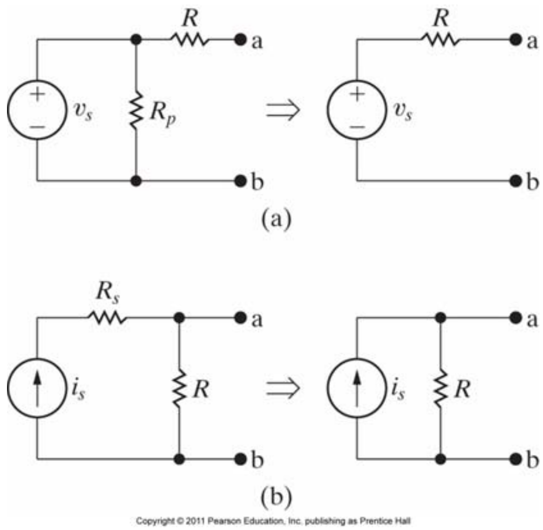
$$i_s = \frac{v_s}{R}$$

Why?

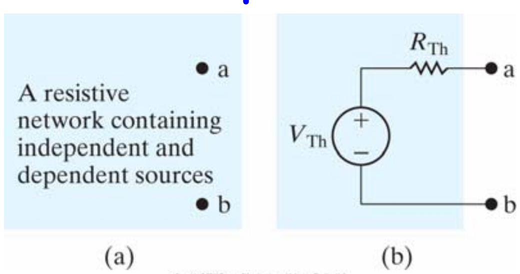




Are these 2 circuits equivalent? Power lost in R_p?



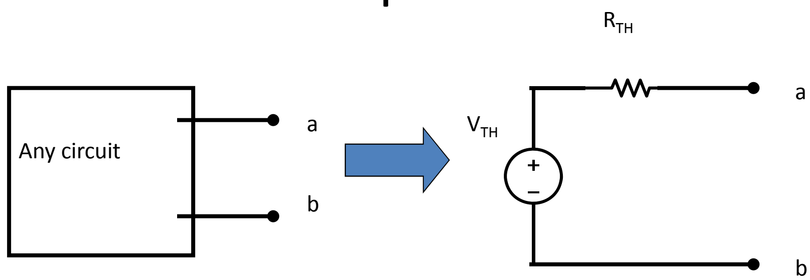
Thevenin Equivalent Circuit



Especially useful when we are interested in the behavior of the circuit between two terminals.

Any resistive network with dependent or independent sources could be represented as a dependent voltage source V_{TH} in series with a resistance R_{TH}.

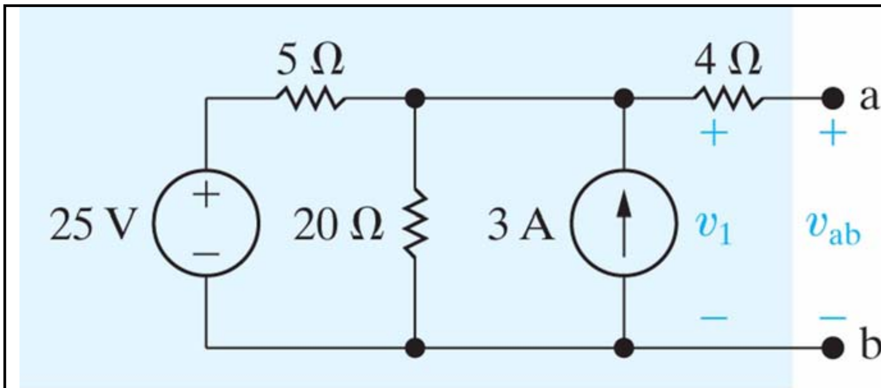
Thevenin equivalent Circuit



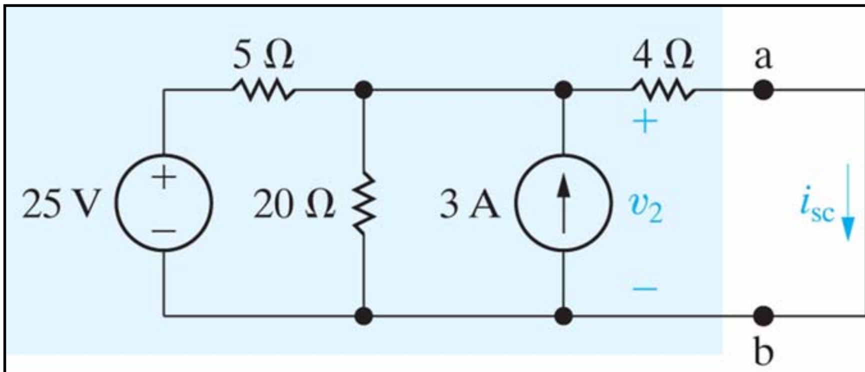
V_{TH} = the open circuit voltage between a and b

$R_{TH} = V_{TH}/I_{sc}$ where I_{sc} is the short circuit current fro a to b

EXCEPT for an ideal current source

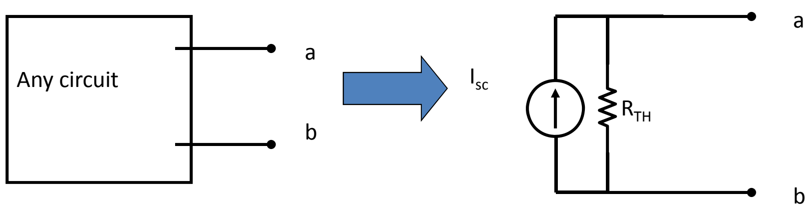


Find Thevenin equivalent



Now find the short circuit current

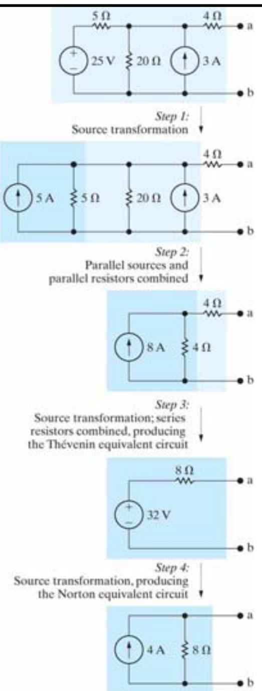
Norton Equivalent Circuit

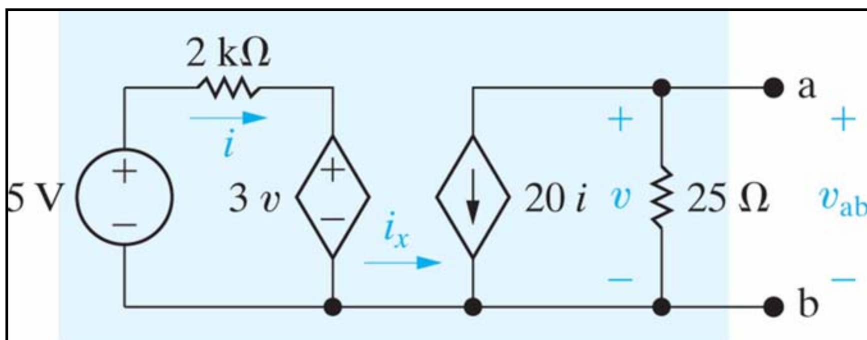


I_{sc} = the short circuit current between a and b

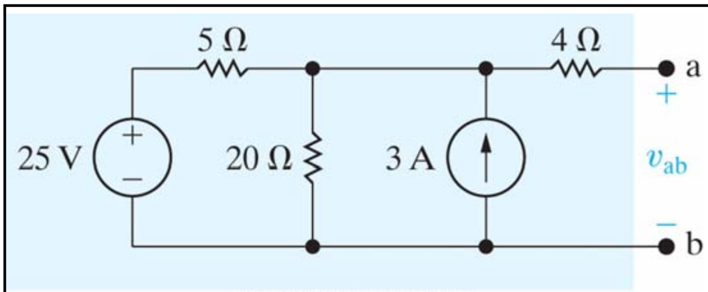
$R_{TH} = V_{TH}/I_{sc}$ where I_{sc} is the short circuit current fro a to b

EXCEPT for an ideal voltage source

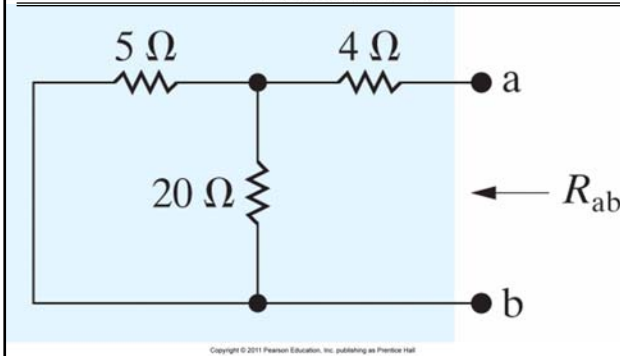




1. Note that ix = 0 (why?)
2. KCL at node a
3. KVL at the loop to the left get $v_{ab} = V_{th}$
4. Short circuit a and b
5. I_{sc}=-20 i, find i from KVL o the left



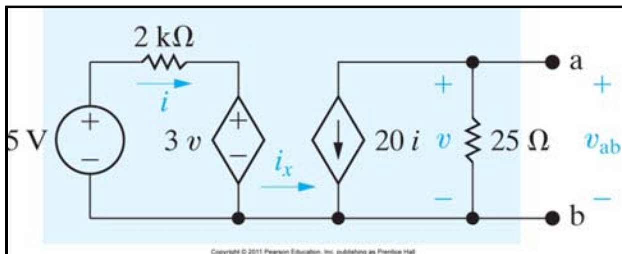
Another wat to calculate R_{TH}



Deactivate all independent sources and calculate R_{ab}

Voltage = short ct.

Current = open ct.



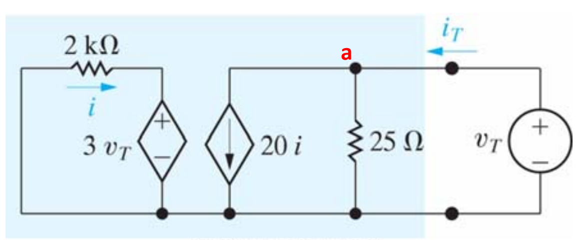
A third way to calculate R_{TH}

Deactivate all independent sources

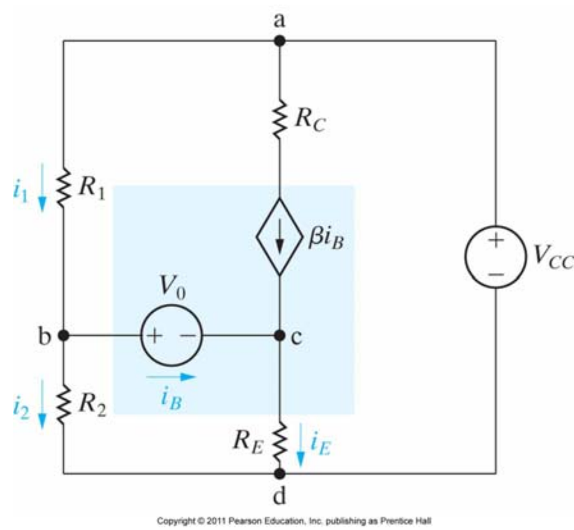
Apply a test voltage (or current) from ab

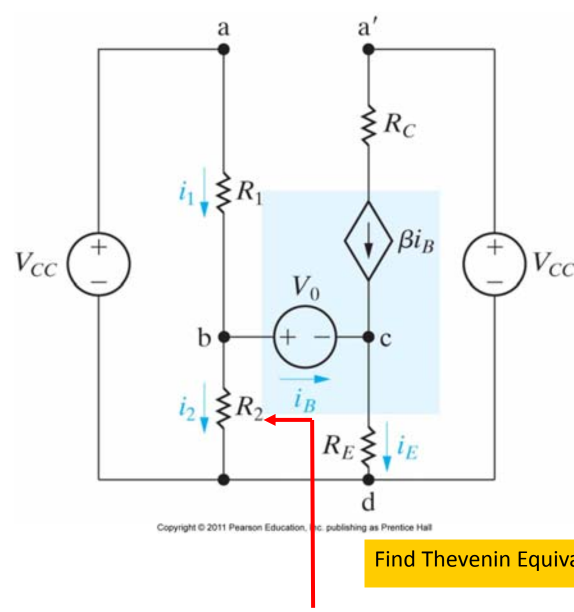
Calculate the current i_T

$$R_{TH} = v_T / i_T$$

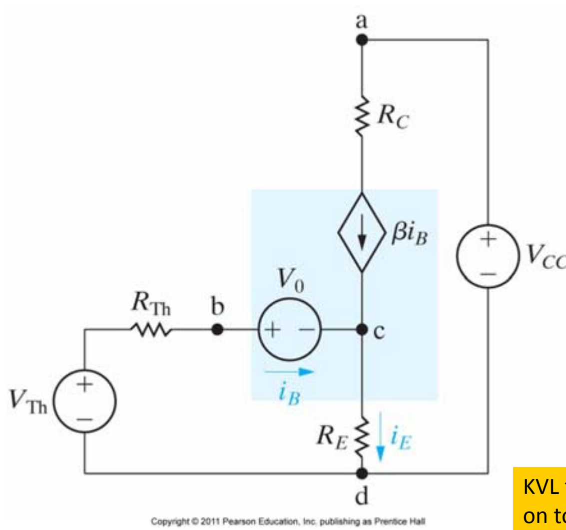


Redraw





Find Thevenin Equivalent



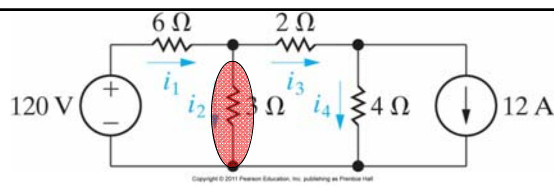
KVL to the small loop on to left, find i_B

Maximum Power Transfer

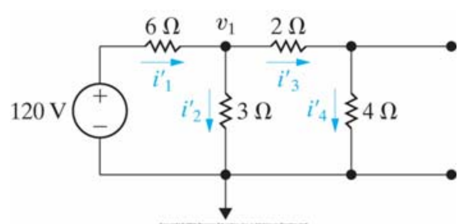
- Assume a voltage source, V with internal resistance R_0 is driving a load R_L
- What is the maximum power R_L can get?
- Calculate p(R_L,R_0) and differentiate =0
- What if it is not a simple source with an internal resistance?
- Thevenin equivalent

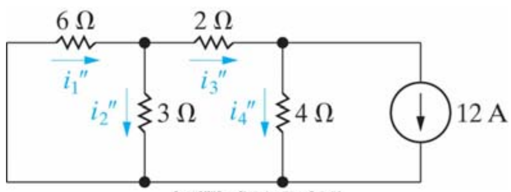
Superposition

- When a linear system is excited by more than one independent source, the total response is the sum of the individual responses where each response is the result of one of the independent sources acting alone.

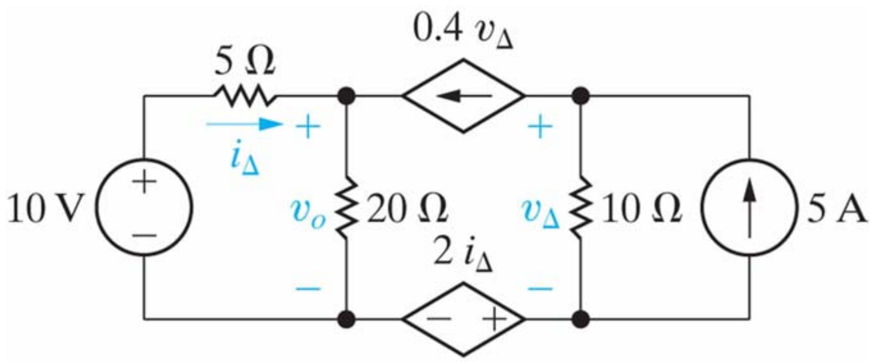


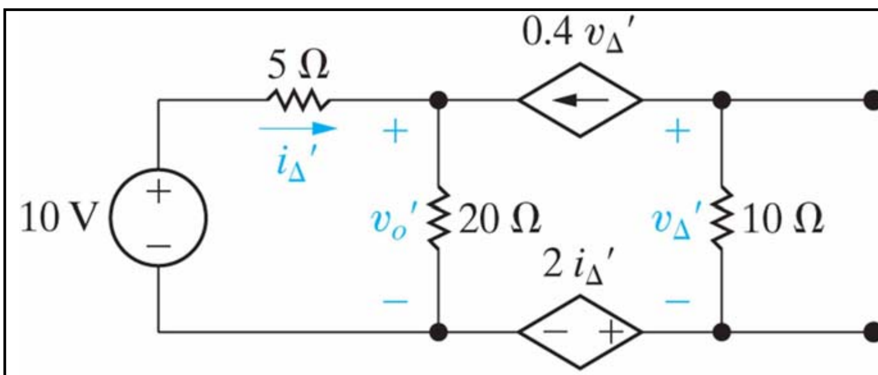
Find the current here

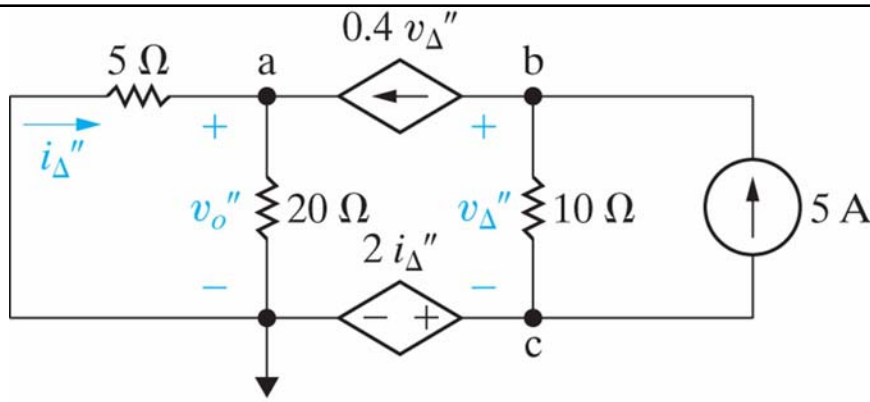




Find v_o using superposition







0.4 v_Δ''
5 Ω
a
b
+
+
i_Δ''
v_o''
20 Ω
v_Δ''
10 Ω
5 A
2 i_Δ''
−
−
− +
c