ENG2200
Electric Circuits

Chapter 5
Inductance, Capacitance and Mutual
Inductance

Objectives

- Understand how an inductor or a capacitor behaves in the presence of a constant current or constant voltage
- Know and being able to use the equations of current, voltage power and energy in an inductor and capacitor
- Be able to combine inductors (capacitors) with an initial condition in series and parallel to form a single equivalent conductor (capacitor).
- Understand the basic concept of mutual inductance and being able to write mesh-current equations for a circuit containing magnetically coupled coils using the dot convention correctly.
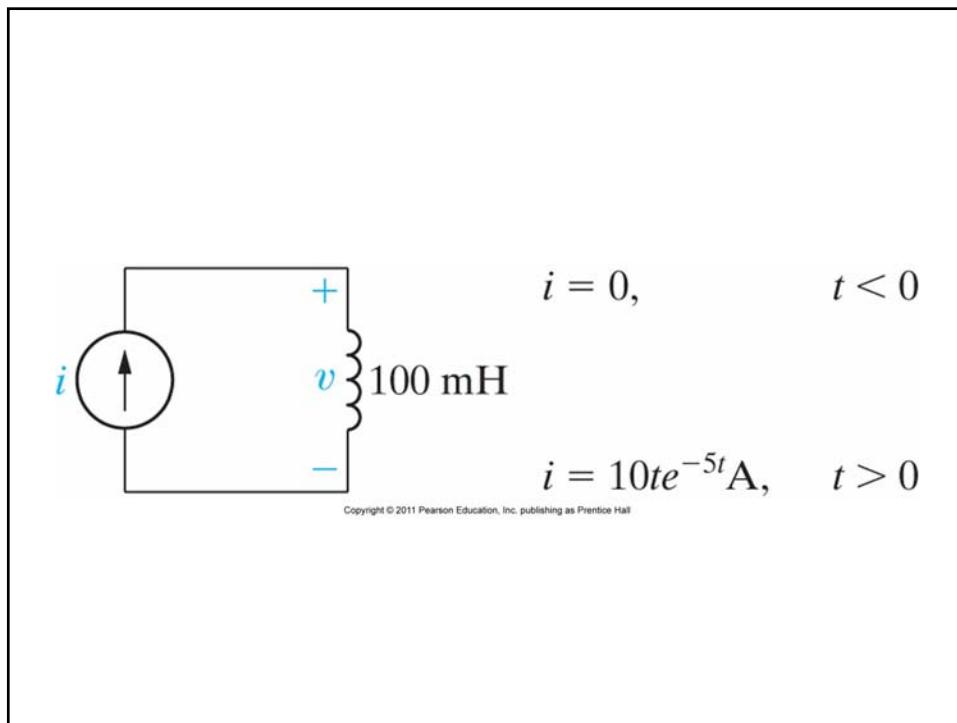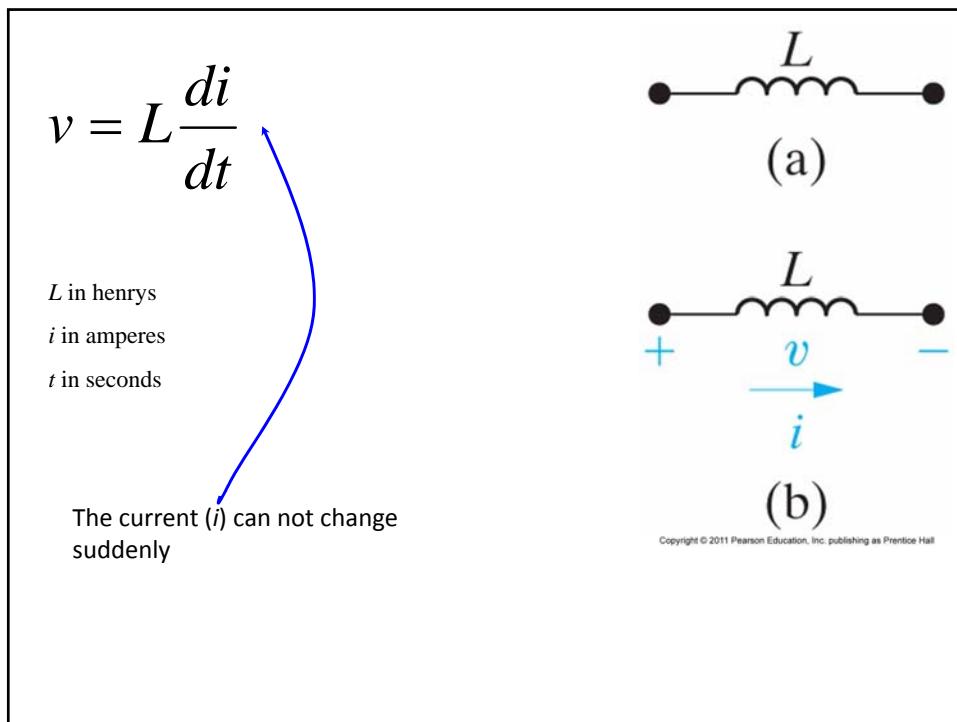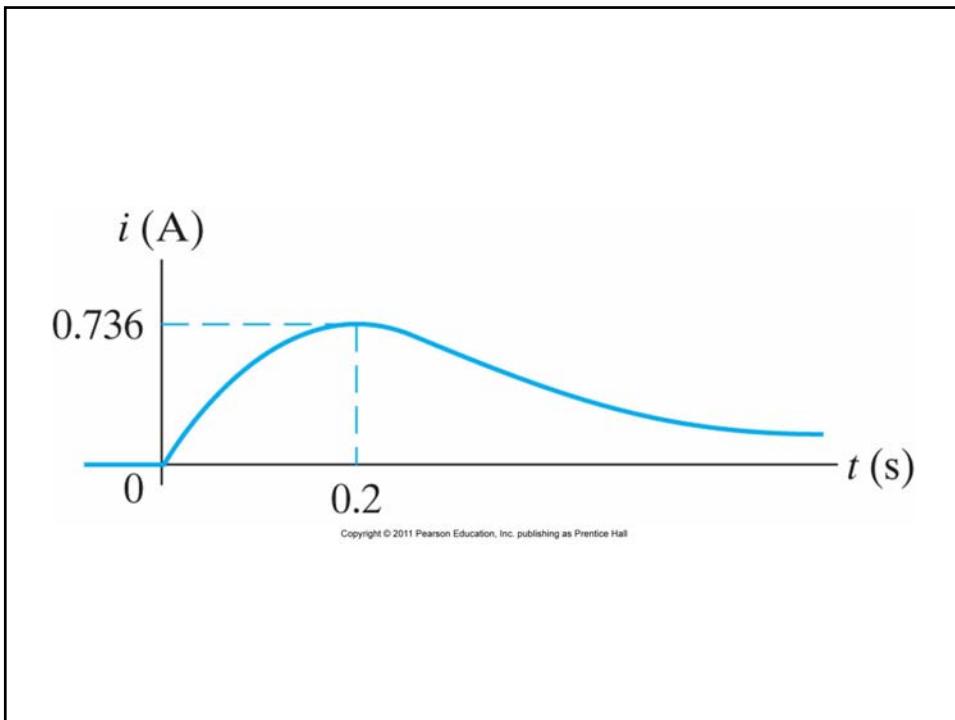
$$v = L\frac{di}{dt}$$

L in henrys

i in amperes

t in seconds

The current (i) can not change suddenly

L

(a)

L

$+$ v $-$

i

(b)

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i $+$ v 100 mH $-$

$i = 0,$ $t < 0$

$i = 10te^{-5t}\text{A},$ $t > 0$

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2

i (A)

0.736

0

0.2

t (s)

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$$i = \begin{cases} 0 & t < 0 \\ 10te^{-5t}\,\mathrm{A} \end{cases}$$

v (V)

$$v = L\frac{di}{dt}$$

1.0

$$v = 0.1\left(-50te^{-5t} + 10e^{-5t}\right)$$

0

0.2

0.6

t (s)
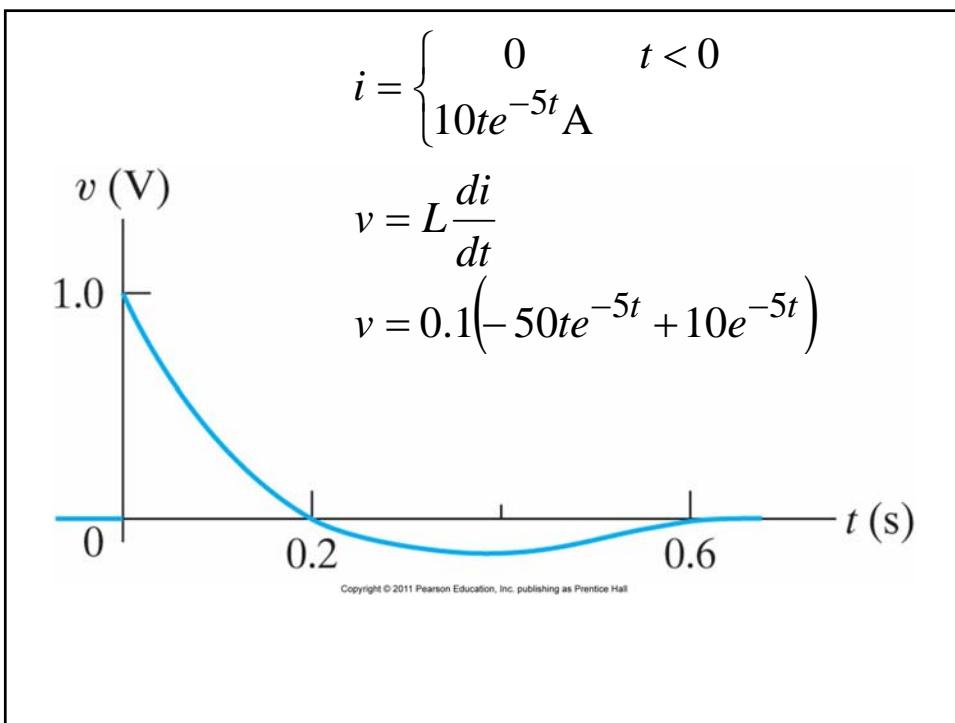
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Inductor

$$v = L\frac{di}{dt}$$

$$vdt = Ldi$$

$$L\int_{i(t_0)}^{i(t)} dx = \int_0^t vdt$$

$$i(t) = \frac{1}{L}\int_0^t vd\tau + i(t_0)$$

$$v = 0, \qquad\qquad t < 0$$

$$v = 20te^{-10t}\,\text{V}, \quad t > 0$$

v (V)

0.736

t (s)

0   0.1   0.2   0.3

i (A)

2

1

0   0.1   0.2   0.3 t (s)

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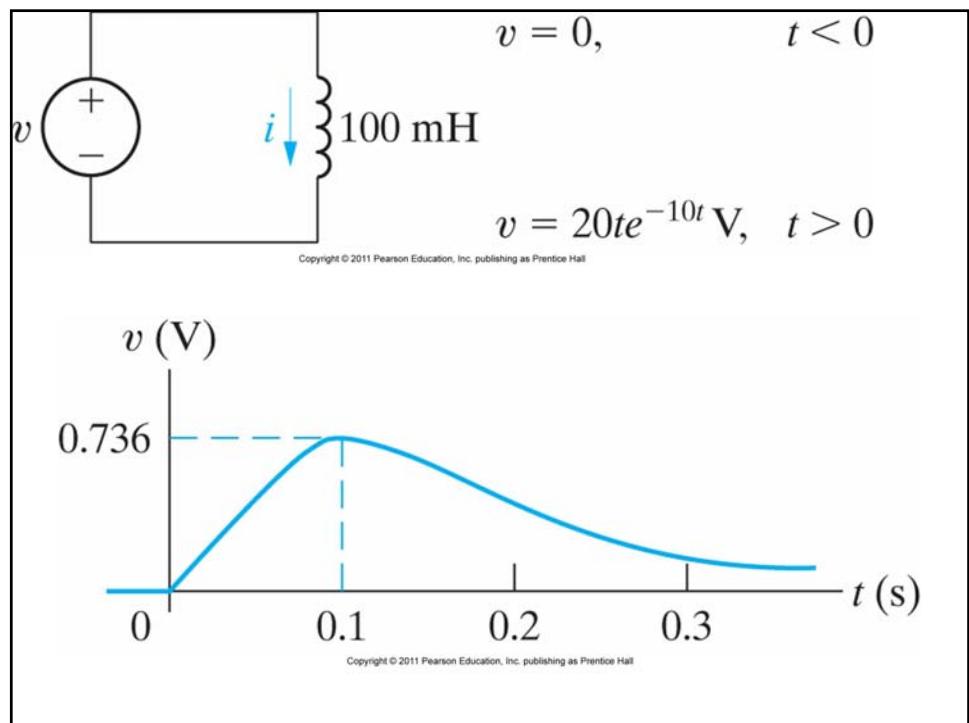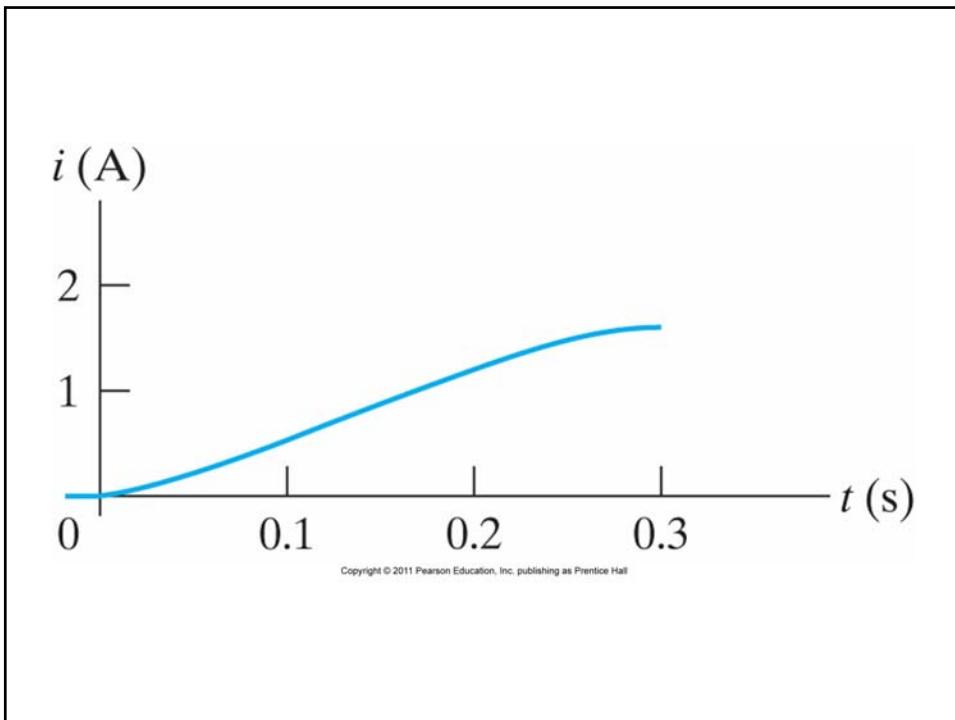
Power and Energy

- Power in an inductor can be expressed as p=vi
- $p = Li\frac{di}{dt}$
- $p = v\left[\frac{1}{L}\int_{t_0}^{t} vd\tau + i(t_0)\right]$
- $p = \frac{dw}{dt} = Li\frac{di}{dt} \rightarrow dw = Ldi$
- $\int_0^w dx = L\int_0^i ydy$
- $w = \frac{1}{2}Li^2$

$i(t) = 10te^{-5t}A$, $L= 0.1H$

$v = e^{-5t}(1 - 5t)V$

$p = vi = 10te^{-10t} - 50t^2e^{-10t}W$

$$\int_0^{0.2} pdt = 27.07 \; mJ$$

$$\int_{0.2}^{\infty} pdt = -27.07 \; mJ$$
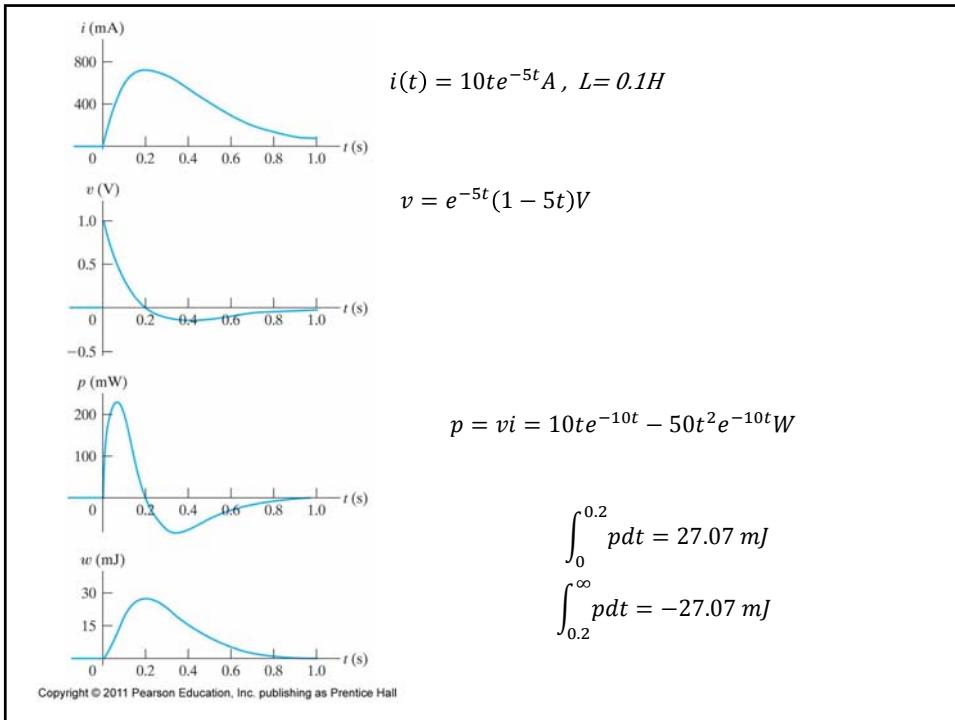
Capacitors

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} id\tau + v(t_0)$$

$$p = i\left[\frac{1}{C}\int_{t_0}^{t} id\tau + v(t_0)\right]$$

$$w = \frac{1}{2}Cv^2$$

(a)

(b)

$$v(t) = \begin{cases} 0 & t \le 0 \\ 4t \ V & 0 \le t \le 1 \\ 4e^{-(t-1)}V & t \ge 1 \end{cases}$$
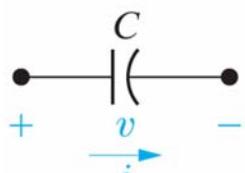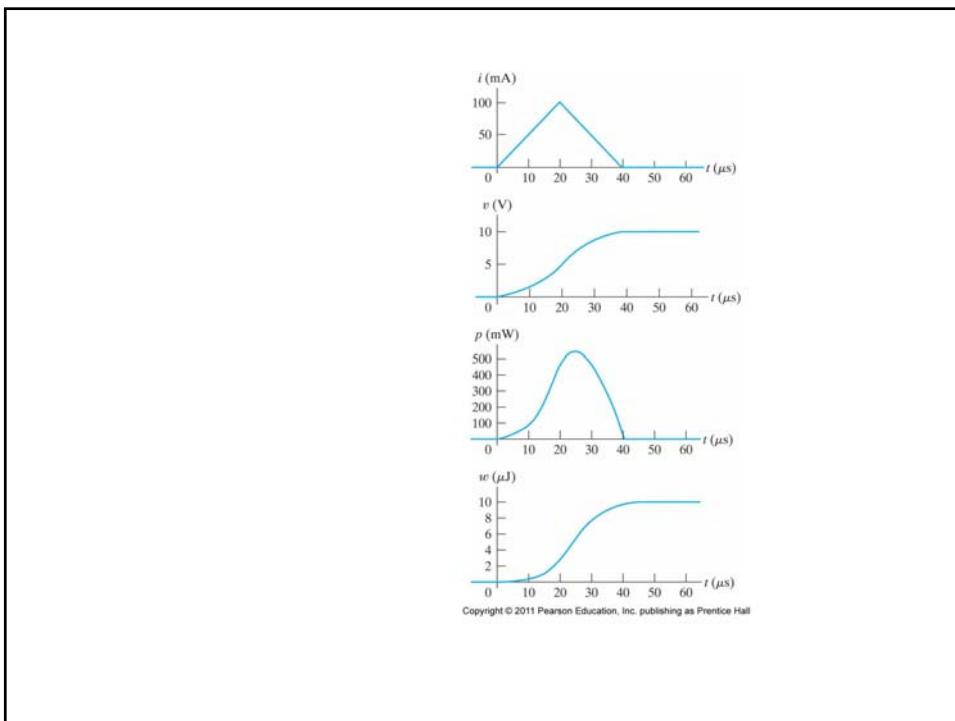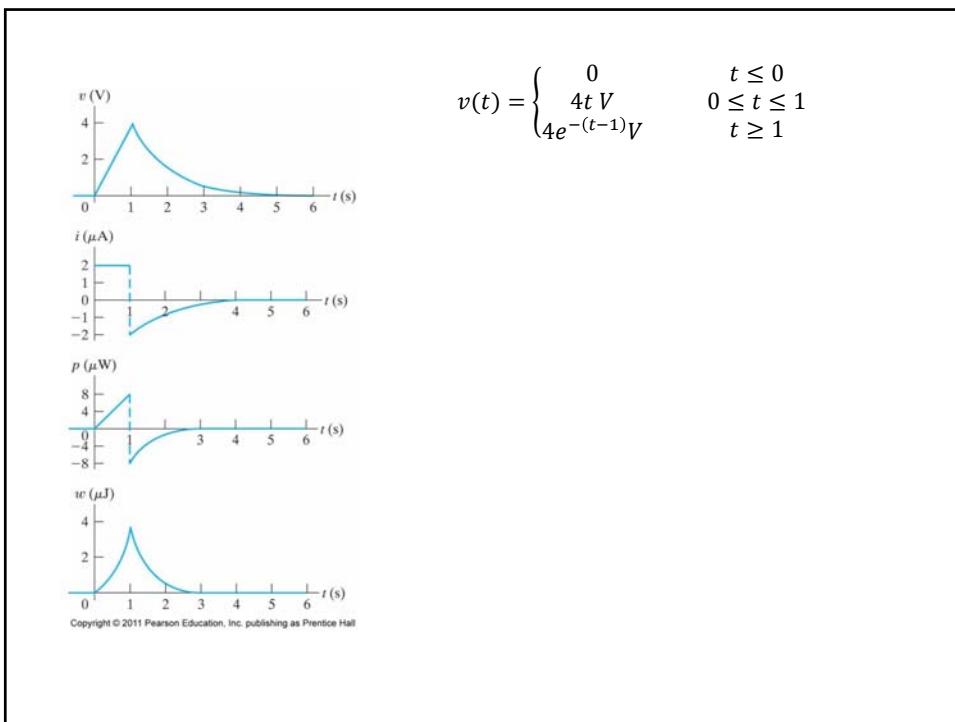
Figure 6.13 Inductors in series.
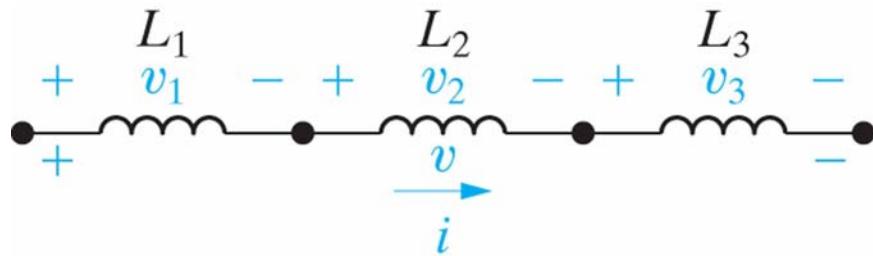
$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$v = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

$i = i_1 + i_2 + i_3$

$$i(t) = \frac{1}{L_1}\int_0^t v\,d\tau + i_1(t_0) + \frac{1}{L_2}\int_0^t v\,d\tau + i_2(t_0) + \frac{1}{L_3}\int_0^t v\,d\tau + i_3(t_0)$$

$$i(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_0^t v\,d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$
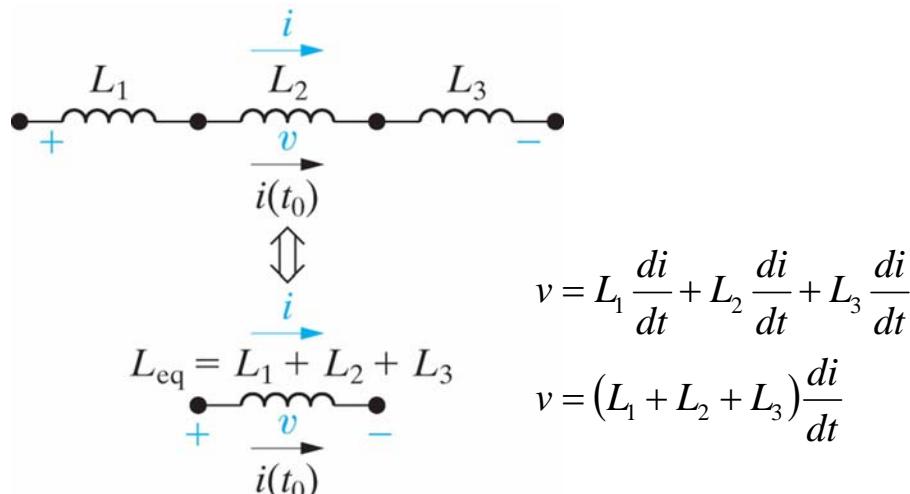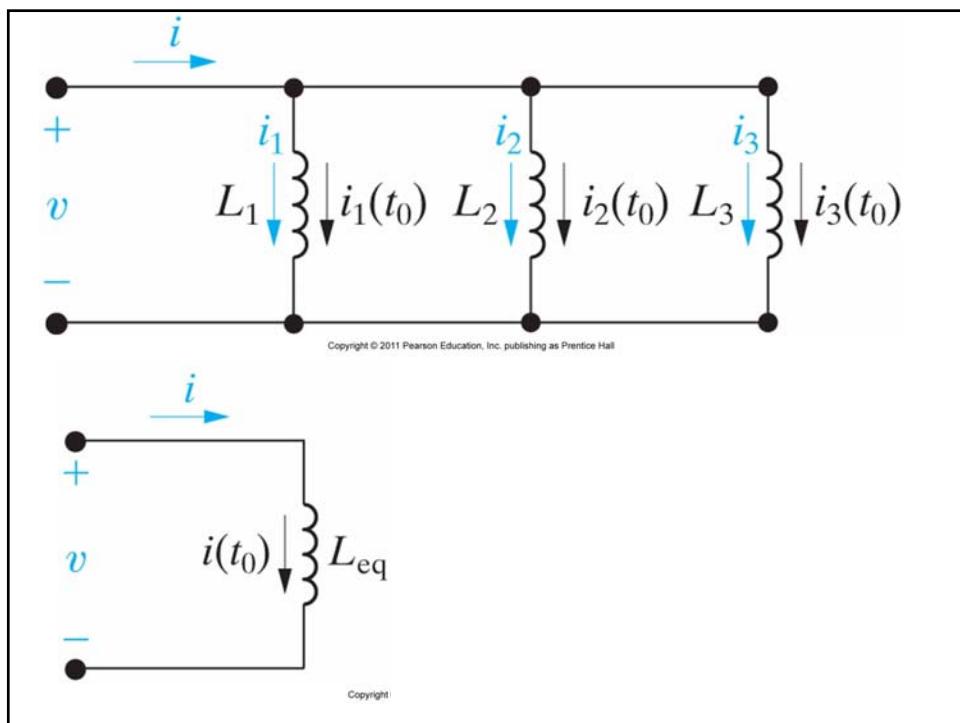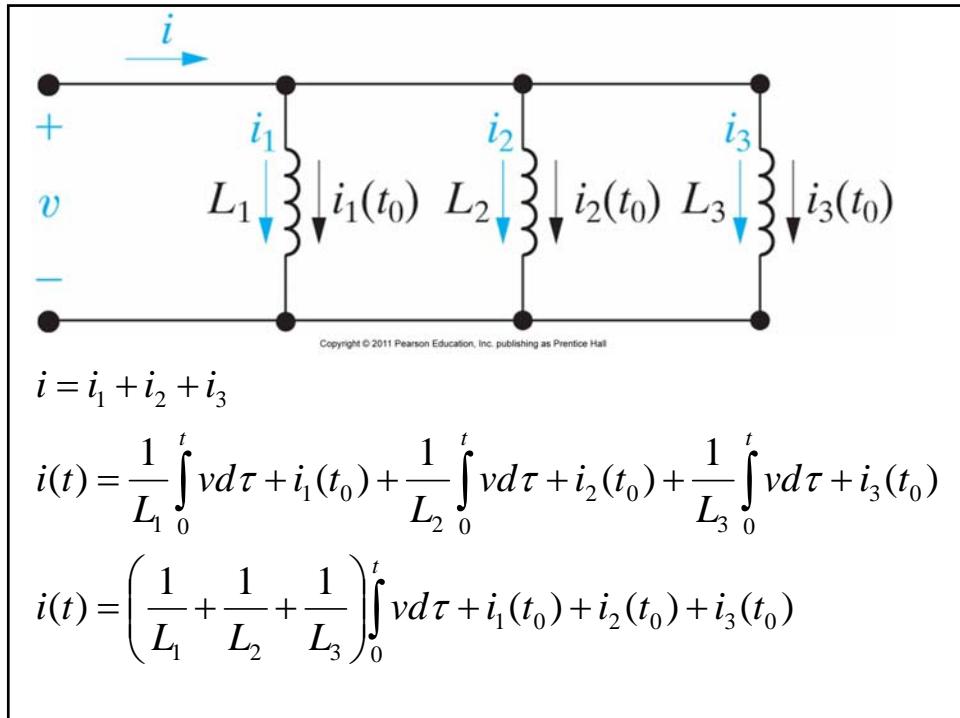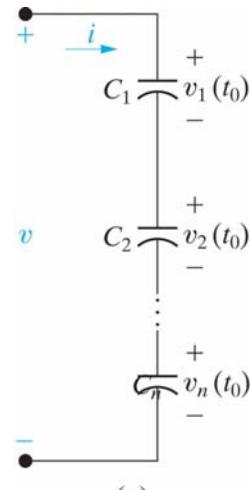
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

(a)   (b)

(a)

$$C_{eq} = C_1 + C_2 + \cdots + C_n$$

(b)
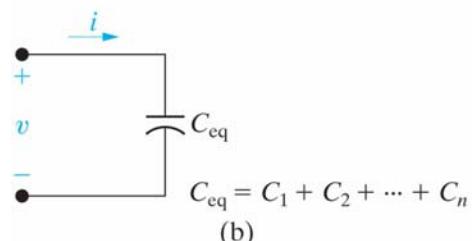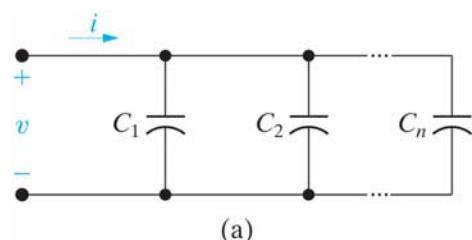
Mutual Inductance

- So far, we are studying a single circuit element, where the change in the current affects the voltage across that element (and vise versa).

- Now, we consider two circuits linked by a magnetic field, such that changes in the current in the first circuit affect the voltage in the second circuit.
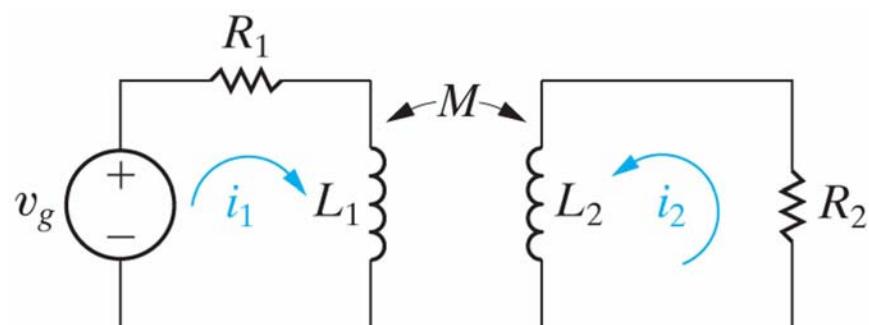
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Dot Convention: When the reference direction for a current entering (leaving) the dotted terminal in a coil, the reference polarity of the voltage that it induces in the other coil is positive (negative) at its dotted terminal

$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

$$i_2 R_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$
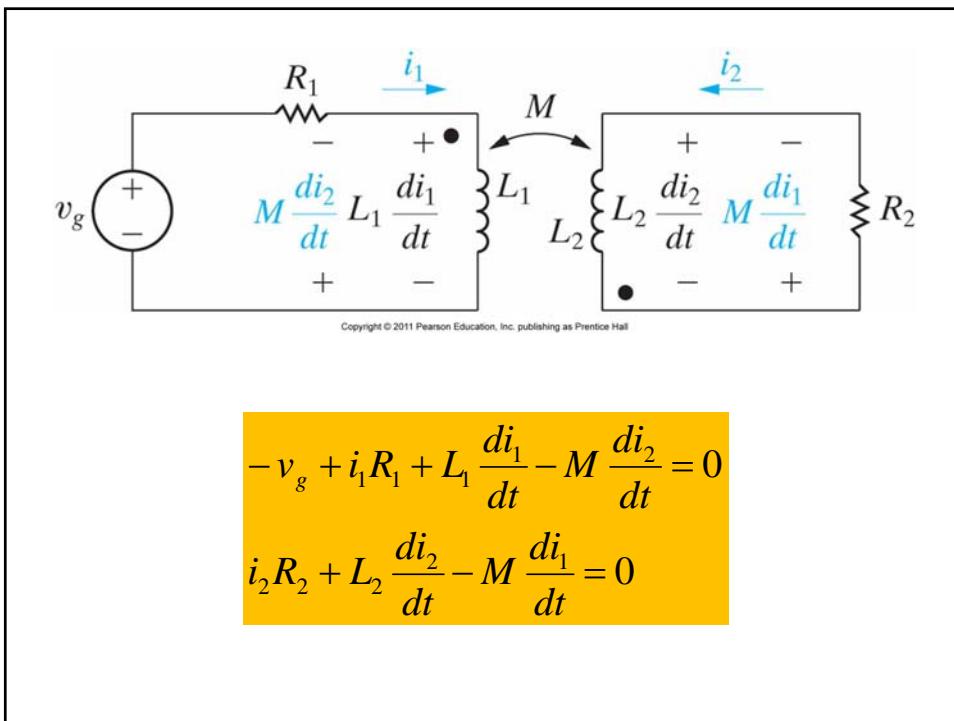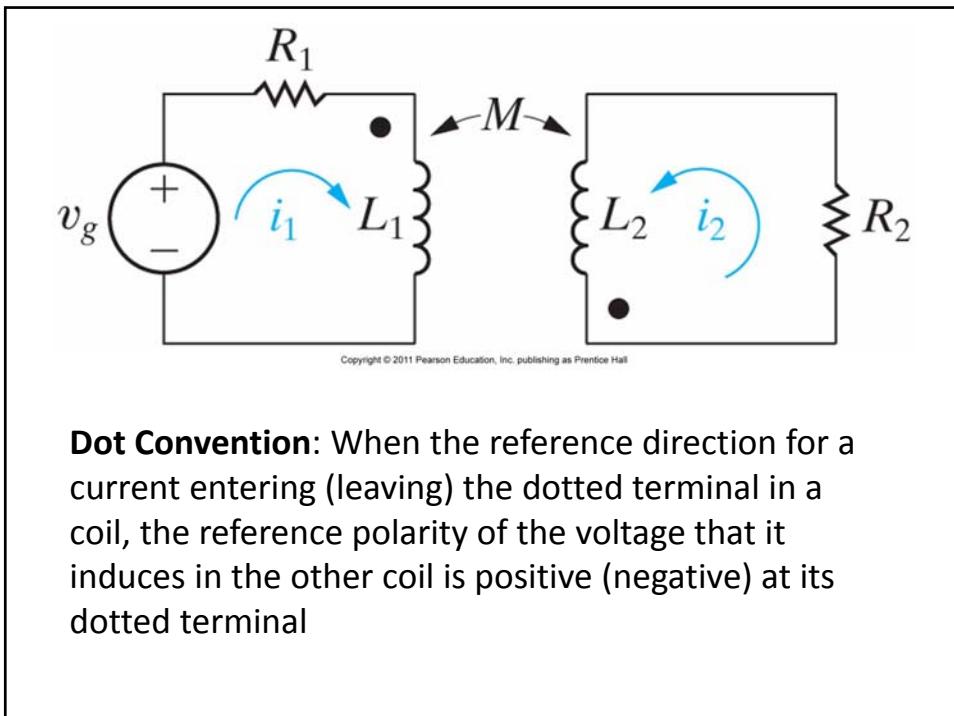
How to determine Polarity
1- you know how coils are wound

1. Arbitrarily mark one terminal of one coil (D)
2. Assign a current into that terminal (i_D)
3. Use the right-hand rule to determine the direction of the magnetic flux inside the coils
4. Arbitrarily pick one terminal (A) of the other coil and assign a current into it (i_A)
5. Use the right-hand rule to determine the direction of the magnetic flux from this current inside the coils
6. Compare the two fluxes directions: If the direction is the same place a dot at the referenced terminal (A) if they are opposite select the opposite terminal (B).

(Step 5)

C

A

i_A

(Step 4)

ϕ_A

(Step 2)

ϕ_D i_D

B   (Step 3)

Arbitrarily dotted terminal (Step 1)

D

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R

Switch

V_{BB}

+

−

dc voltmeter

+

−

How to determine Polarity
2- you do not know how coils are wound

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When the switch is closed...
- The second polarity mark goes on the terminal connected to the **positive terminal** of the voltmeter if the voltmeter momentarily deflects **upscale**
- The second polarity mark goes on the terminal connected to the **negative terminal** of the voltmeter if the voltmeter momentarily deflects **downscale**

$$4\frac{di_1}{dt}+8\frac{d}{dt}(i_g-i_2)+20(i_1-i_2)+5(i_1-i_g)=0$$

$$20(i_2-i_1)+60i_2+16\frac{d}{dt}(i_g-i_2)-8\frac{di_1}{dt}=0$$
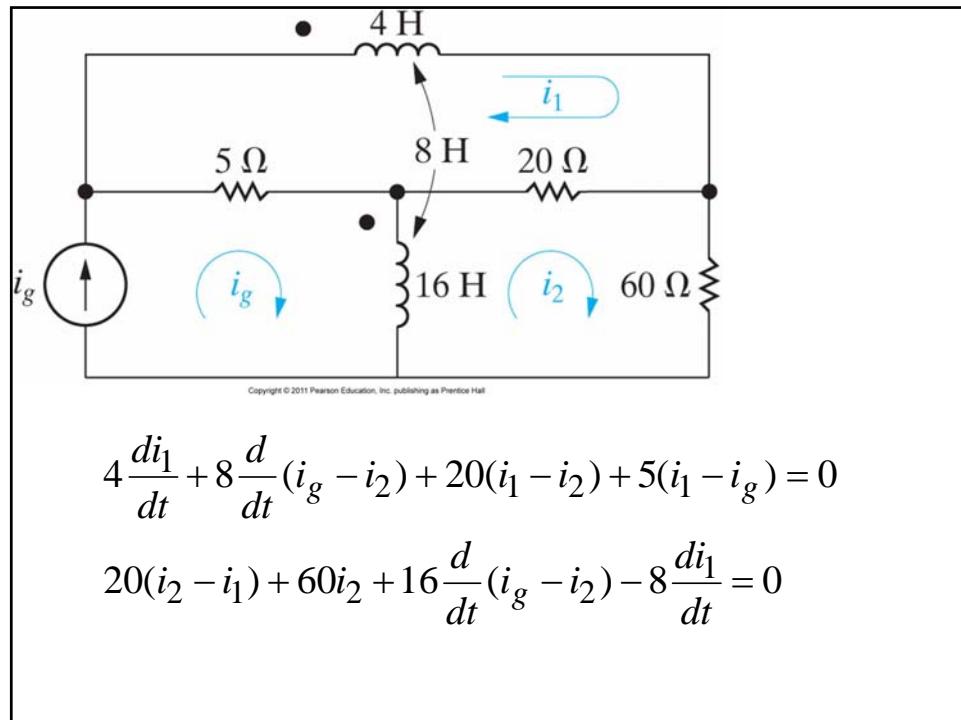
Figure 6.27 Representation of a magnetic field linking an N-turn coil.
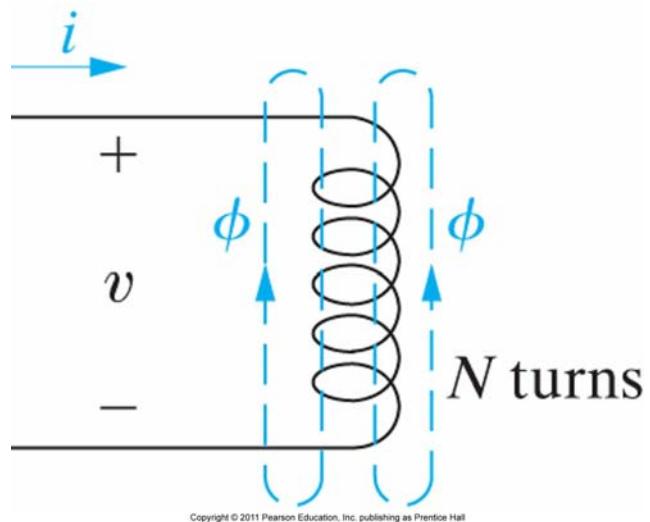
**Figure 6.28** Two magnetically coupled coils.
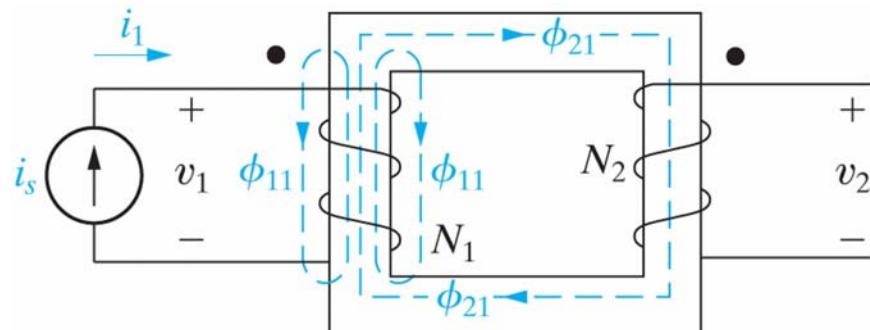
Figure 6.29 The magnetically coupled coils of Fig. 6.28, with coil 2 excited and coil 1 open.