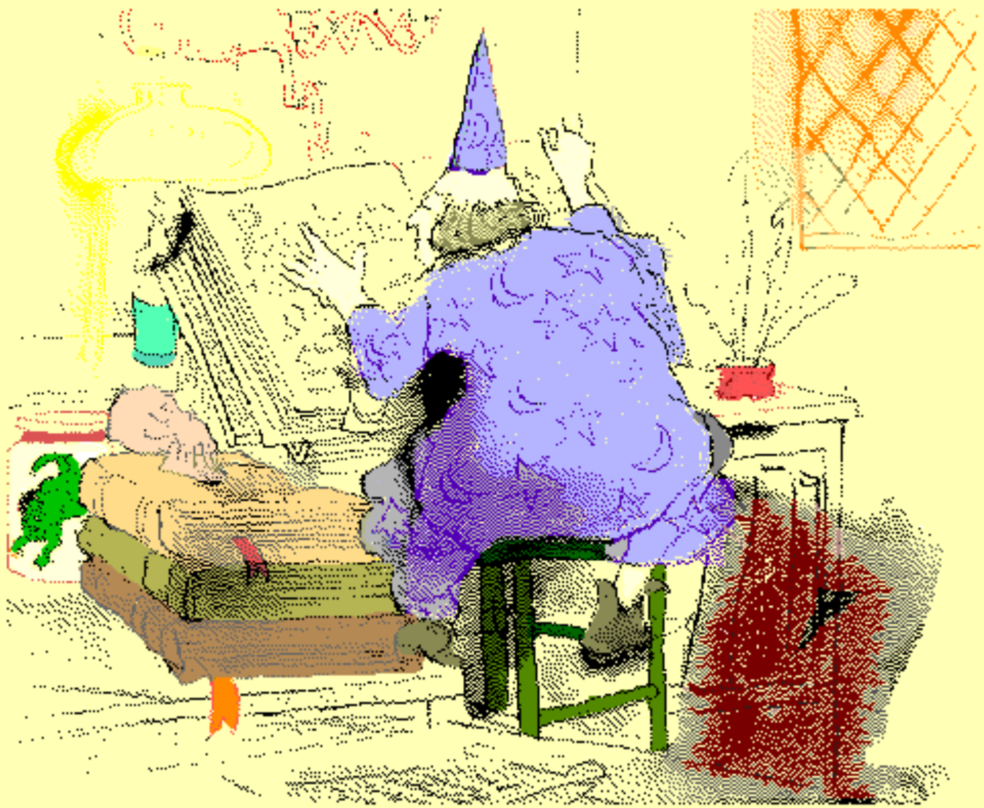
# Introductrion

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## Introduction to Soft Computing

### Introduction to Logic, to Fuzzy Sets, to Fuzzy Logic

# Soft Computing: The Course

Everything you need is on the course WIKI: [https://wiki.cse.yorku.ca/](https://wiki.cse.yorku.ca/)

The course will be sectioned into parts:

- Part I – Fuzzy Sets and Fuzzy Logic
- Part II – Rough Sets
- Part III – Neural Networks
- Part IV – Evolutionary Computing
- Part V – Probabilistic Reasoning
- Part VI – Applications, Intelligent Systems design, Hybrid Systems
- Part VII – Student Presentations

- Parts I-VI will primarily rely on instructor lectures with significant student involvement; Part VII will rely on students making presentations.

# Soft Computing: The Course

Learning expectations include:

- Knowledge of the terminology and concepts of soft computing;
- Insight into the possibilities and fundamental limitations of soft computing;
- Insight into the relative advantages and disadvantages of the major approaches to soft computing (fuzzy sets, rough sets. Evolutionary computing, neural networks, probabilistic reasoning and so on);
- Understanding of the basic methods and techniques used in soft computing;
- Skills in applying the basic methods and techniques to concrete problems in soft computing.

# Soft Computing: The Course

Grading – Undergraduates:

- The course will be graded on the basis of one minor and substantial assignment (10% and 15%), one in-class presentation (10%), one final exam (25%) and one project (40%).

Grading – Graduate Students:

- The course will be graded on the basis of one minor and substantial assignment (10% and 15%), one in-class presentation (10%), one final exam (10%), one paper(20%) and one project (40%).

# Logic

Read the first 10 pages of the handout on logic for a refresher on propositional calculus and predicate logic.

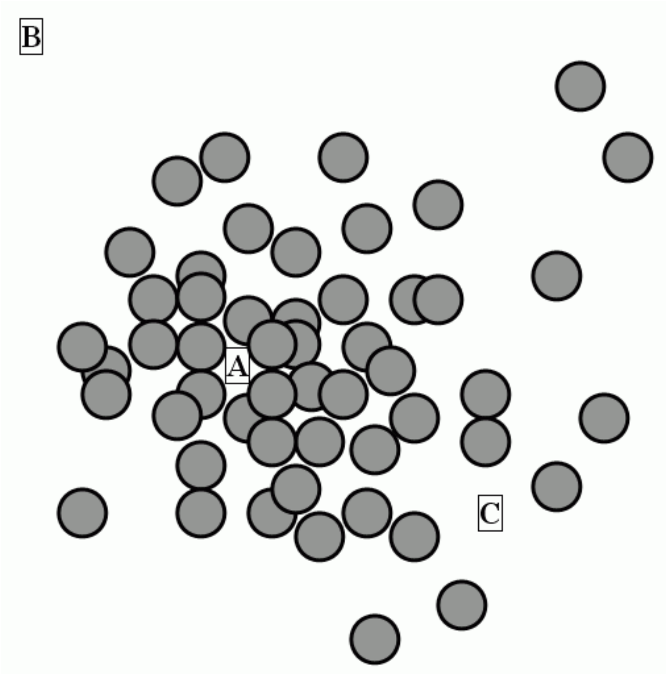
# Fuzzy Sets

- Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets were introduced by Lotfi Zadeh (1965) as an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition — an element either belongs or does not belong to the set.

- Fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1]. Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. In fuzzy set theory, classical bivalent sets are usually called *crisp* sets. The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise, such as bioinformatics.

# Fuzzy Sets

Fuzzy sets can be applied, for example, to the field of genealogical research. When an individual is searching in vital records such as birth records for possible ancestors, the researcher must contend with a number of issues that could be encapsulated in a membership function. Looking for an ancestor named John Henry Pittman, who you *think* was born in (probably eastern) Tennessee circa 1853 (based on statements of his age in later censuses, and a marriage record in Knoxville), what is the likelihood that a particular birth record for "John Pittman" is *your* John Pittman? What about a record in a different part of Tennessee for "J.H. Pittman" in 1851? (It has been suggested by Thayer Watkins that Zadeh's ethnicity is an example of a fuzzy set!)




Bird's-eye view on a forest: Where is the boundary of the forest? Which location is in the forest and which is out of it? )

Consider the bird's-eye view of a forest in the figure to the left.

Is location A in the forest? Certainly yes.
Is location B in the forest? Certainly not.
Is location C in the forest? Maybe yes, maybe not. It depends on a subjective (vague) opinion about the sense of the word "forest".

# Fuzzy Sets

- Definition:
- A fuzzy set is a pair ($A$,$m$) where $A$ is a set and A → [0.1].
- For each $x \in$ A, $m(x)$ is called the grade of membership of $x$ in ($A$,$m$). For a finite set $A = \{x_1,...,x_n\}$, the fuzzy set ($A$,$m$) is often denoted by $\{m(x_1) / x_1,...,m(x_n) / x_n\}$.

- Let $x \in$ A. Then $x$ is called not included in the fuzzy set ($A$,$m$) if $m(x) = 0$, $x$ is called fully included if $m(x) = 1$, and $x$ is called fuzzy member if $0 < m(x) < 1$. The set is called the support of ($A$,$m$) and the set is called its kernel.

- Sometimes, more general variants of the notion of fuzzy set are used, with membership functions taking values in a (fixed or variable) algebra or structure $L$ of a given kind.

# Fuzzy Sets

- Operations with fuzzy sets

- The processing of fuzzy sets generalizes the processing of the deterministic sets. Namely, if A, B are fuzzy sets with membership functions $\mu_A$, $\mu_B$, respectively, then also the complement ~A, union $A \cup B$, and intersection $A \cap B$ are fuzzy sets, and their membership functions are defined for $x \in U$ *by*

- $\mu_{\sim A}(x) = 1 - \mu_A(x)$

- $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
-
- $\mu_{A \cap B}(x) = \mathrm{m}(\mu_A(x), \mu_B(x))$

# Fuzzy Sets

- Operations with fuzzy sets

- Moreover, the concept of inclusion of fuzzy sets, $A \subset B$, is defined by
- $\mu_A(x) \leq \mu_B(x)$ for all $x \in U$, and the empty and universal fuzzy sets, $\Theta$ and $U$, are defined by membership function $\mu_\Theta(x) = 0$ and $\mu_U(x) = 1$
- for all $x \in U$.

- Even if all above operations and concepts consequently generalize their counterparts in the deterministic set theory, the resulting properties of fuzziness need not be identical with those of the deterministic theory, e.g., for some fuzzy set A, the relation $A \cap \sim A \neq 0$, or even $A \subset \sim A$, may be fulfilled.

# Fuzzy Sets

Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than accurate. In contrast with "crisp logic", where binary sets have binary logic, fuzzy logic variables may have a truth value that ranges between 0 and 1 and is not constrained to the two truth values of classic propositional logic. Furthermore, when linguistic variables are used, these degrees may be managed by specific functions.

Fuzzy logic emerged as a consequence of the 1965 proposal of fuzzy set theory by Lotfi Zadeh. Though fuzzy logic has been applied to many fields, from control theory to artificial intelligence, it still remains controversial among most statisticians, who prefer Bayesian logic, and some control engineers, who prefer traditional two-valued logic.

# Fuzzy Logic

Fuzzy logic and probabilistic logic are mathematically similar – both have truth values ranging between 0 and 1 – but conceptually distinct, due to different interpretations. Fuzzy logic corresponds to "degrees of truth", while probabilistic logic corresponds to "probability, likelihood"; as these differ, fuzzy logic and probabilistic logic yield different models of the same real-world situations.

Both degrees of truth and probabilities range between [0, 1] and may seem similar at first. For example, let a 100 ml glass contain 30 ml of water. We may consider two concepts: Empty and Full. The meaning of each of them can be represented by fuzzy set. One might define the glass as being 0.7 empty and 0.3 full. Note that the concept of emptiness is subjective. We might equally well design a set membership function where the glass would be considered full for all values down to 50 ml.

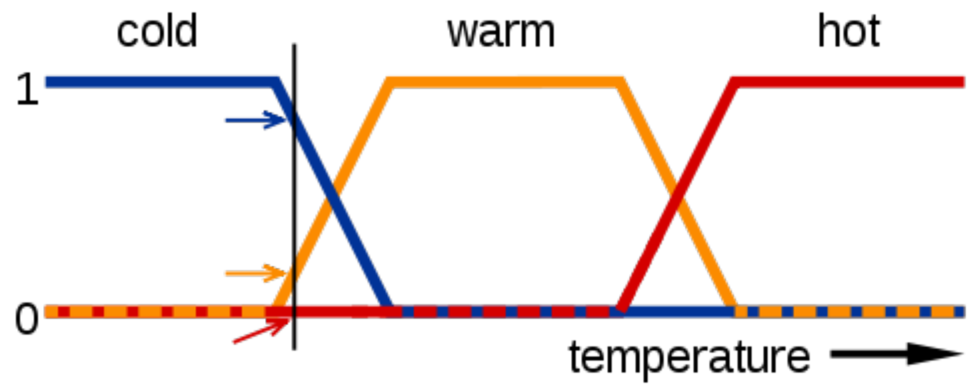
# Fuzzy Logic

It is essential to realize that fuzzy logic uses truth degrees as a mathematical model of the vagueness phenomenon while probability is a mathematical model of ignorance. The same could be achieved using probabilistic methods, by defining a binary variable "full" that depends on a continuous variable that describes how full the glass is. There is no consensus on which method should be preferred in a specific situation

## Applying truth values

A basic application might characterize subranges of a continuous variable. For instance, a temperature measurement for anti-lock brakes might have several separate membership functions defining particular temperature ranges needed to control the brakes properly. Each function maps the same temperature value to a truth value in the 0 to 1 range. These truth values can then be used to determine how the brakes should be controlled.

# Fuzzy Logic - example




Example - Fuzzy logic temperature: In this image, the meaning of the expressions *cold*, *warm*, and *hot* is represented by functions mapping a temperature scale. A point on that scale has three "truth values"—one for each of the three functions. The vertical line in the image represents a particular temperature that the three arrows (truth values) gauge. Since the red arrow points to zero, this temperature may be interpreted as "not hot". The orange arrow (pointing at 0.2) may describe it as "slightly warm" and the blue arrow (pointing at 0.8) "fairly cold".

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## • Concluding Remarks

- The Road to Wisdom

- The road to wisdom? –
- Well, it's plain and simple to express:

- Err and err and err again but less and less and less.