EECS2602.4 Continuous Signals and Systems

No. 2 Time-Domain Analysis of LTIC Systems

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15-01-28 1

CT Systems

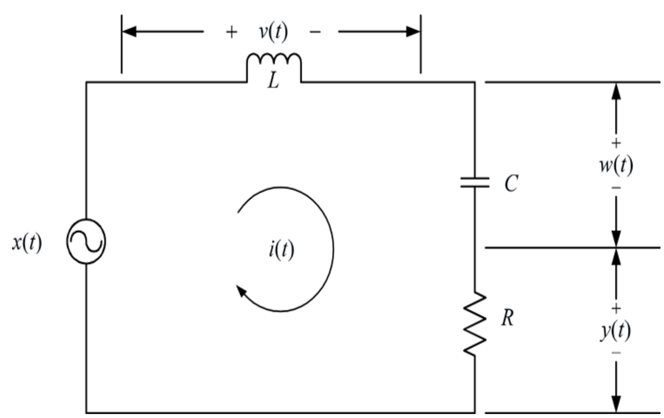
- Analysis
  - Time-domain: x(t) → y(t)
  - Frequency-domain: Fourier transform
  - Laplace transform
- Design
  - Continuous-time filter design

15-01-28 2

LTIC systems (I)

- LTIC system can be modeled by a linear constant-coefficient differential equation.
- Example: Series RLC circuit
- How to solve a linear constant-coefficient differential equation?
  - Initial conditions are necessary
  - Not all initial conditions lead to an LTIC system

Series RLC Circuit



Linear constant-coefficient differential equation (review)

- Zero-input response: solution to a linear constant coefficient differential equation without input signals.
- Zero-state response: solution to a solution to a linear constant coefficient differential equation with zero inintial conditions.
  - Homogeneous component + Particular component
- Solution to a general linear constant coefficient differential equation is superposition of
  - Homogeneous component + Particular component
- Determine unknown coefficients based on initial conditions

15-01-28 5

Example (I)

- Compute only zero-input reponse.
- No input singal, output signal is generated all by the internal energy stored in the system.
- The system is not strictly linear in this case.

$$\frac{d^2w(t)}{dt^2}+7\frac{dw(t)}{dt}+12w(t)=12x(t) \qquad w(0^-)=2,\ \dot{w}(0^-)=5$$

make $x(t)=0$

15-01-28 6

Example (II)

- "Initially Rest" LTIC systems
- Zero-input reponse is zero (no internally stored energy)
- The system output signals are called the zero-state response
- "Initially Rest" LTIC systems are strictly linear systems

$$\frac{d^2w(t)}{dt^2}+7\frac{dw(t)}{dt}+12w(t)=12x(t) \qquad w(0^-)=0,\ \dot{w}(0^-)=0$$

15-01-28 7

LTIC Example (III)

- Systems are not initially rest (with internal energy)
- Systems have input signals
- Output signals are superposition of zero-input and zero-state responses.

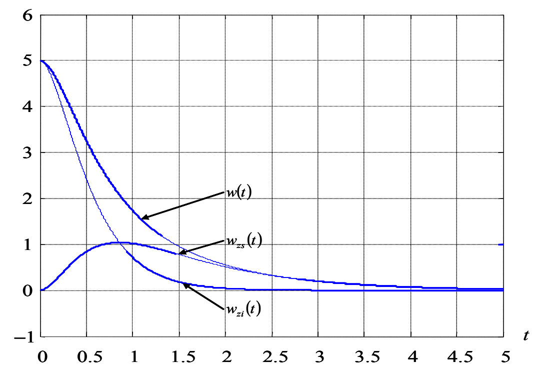
$$\frac{d^2w(t)}{dt^2}+7\frac{dw(t)}{dt}+12w(t)=12x(t) \qquad w(0^-)=5,\ \dot{w}(0^-)=0$$

$x(t)=2e^{-t}u(t)$ or $x(t)=\cos(t)u(t)$

15-01-28 8

LTIC Example (I)

$$\frac{d^2w(t)}{dt^2}+7\frac{dw(t)}{dt}+12w(t)=12x(t) \qquad w(0^-)=5,\ \dot{w}(0^-)=0$$

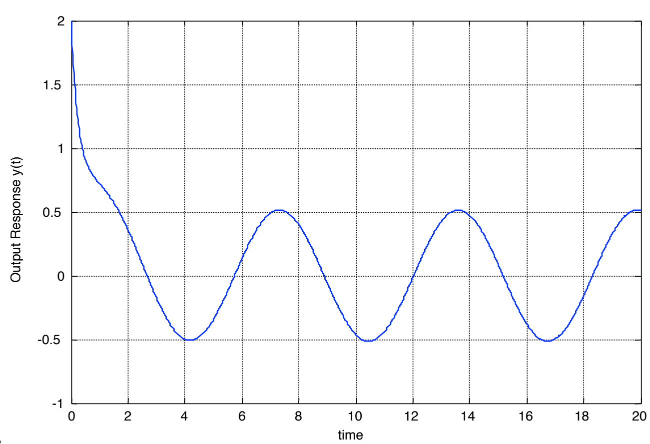


$$x(t)=2e^{-t}u(t)$$

15-01-28 9

LTIC Example (II)

$$\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+4y(t)=3x(t) \qquad y(0^-)=2,\ \dot{y}(0^-)=-5 \qquad x(t)=\cos(t)u(t)$$

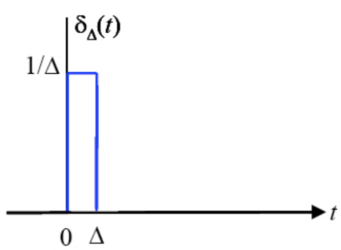


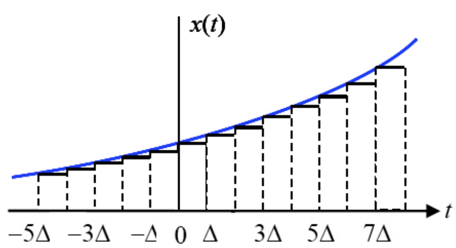
15-01-28 10

LTIC systems: impulse response

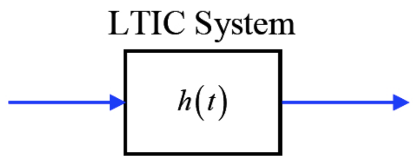
- An LTIC system can be modeled by its impulse response.
- What is impulse response?
- Decompose any CT signal using CT impulse signals.
- Convolution integral.

CT signal Decomposition in time domain





Model LTIC by impulse response

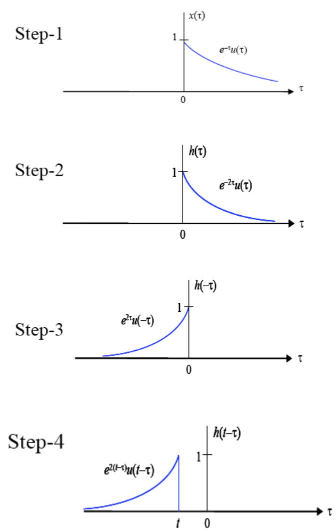


$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

15-01-28 13

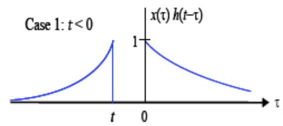
Convolution Integral (I)



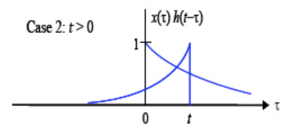
15-01-28 14

Convolution Integral (II)

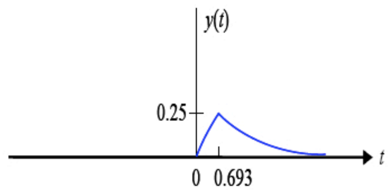
Step-5a



Step-5b



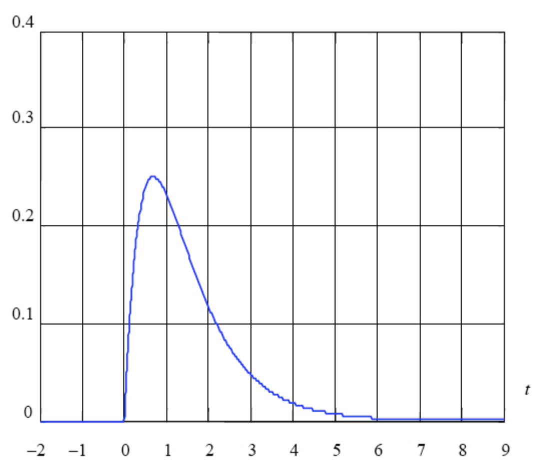
Step-6



15-01-28 15

Convolution Integral (III)

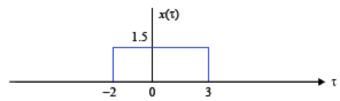
Final result for all t values: y(t) = x(t) * h(t)



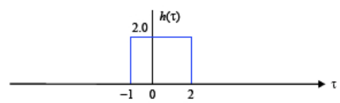
15-01-28 16

Convolution Example

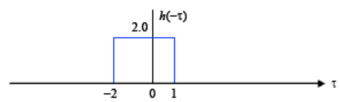
Step-1



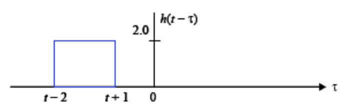
Step-2



Step-3



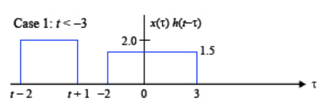
Step-4



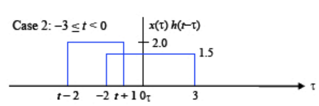
15-01-28 17

Convolution Example

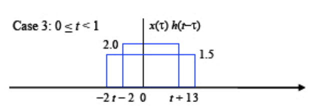
Step-5a



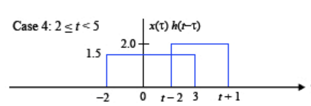
Step-5b



Step-5c



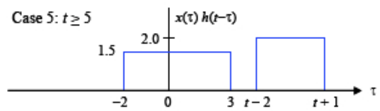
Step-5d



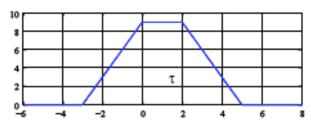
15-01-28 18

Convolution Example

Step-5e



Step-6



15-01-28 19

Properties of Convolution Integral

- Commutative property
- Distributive property
- Associative property
- Shift property
- Duration of convolution
- Convolution with impulse signal
- Convolution with unit step function
- Scaling property

15-01-28 20

Impulse Response of LTIC Systems

- Memoryless LTIC systems
- Causal LTIC systems
- Stable LTIC systems
- Invertible LTIC systems