EECS2602 Z: Continuous Time Signals and Systems
Instructor: Hui Jiang
Quiz # 2 (12% of the course) your mark: / 70
Time Allowed: 60 minutes

Name: ____

Student ID Number: ____________________York EECS Email: ____________________

1. (12 points) Write TRUE or FALSE for each of the following statements and justify briefly.

3.1 The system $y(t) = |x(t)|$ is invertible. [**FALSE**]

$y(t) = |x(t)|$ is not one-to-one mapping function

3.2 The system $y(t) = \text{sgn}(x(t))$ is nonlinear. [**TRUE**]

take one example: x(t) =u(t) → y(t)

take another example: x'(t) = 2 u(t) → y'(t) = y(t) != 2 y(t)

3.3 The system $y(t) = \int_{t-10}^{t} |x(\tau)|\, d\tau$ is non-causal. [**FALSE**]

y(t) only depends on x(t) from t-10 to t, all of which is history …

3.4 The system $\frac{dy(t)}{dt} + 2y(t) = 3x(t)$ is always linear. [**FALSE**]

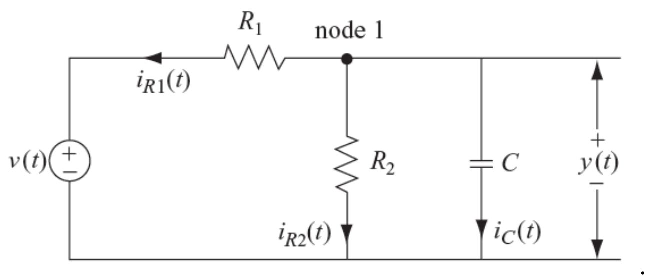
It is not linear if it has non-zero initial conditions.

2. (24 marks) The following electrical circuit consists of two resistors R1 and R2, and a capacitor C.

(i) (8 marks) Determine the differential equation relating the input voltage v(t) to the output voltage y(t).

(ii) (6 marks) Determine whether the system is (a) linear; (b) time-invariant; (c) memory less; (d) causal; (e) invertible; and (f) stable.

(iii) (10 marks) Assume the system is initially rest, R1=1, R2=2, C=1, determine the output signal $y(t)$ when the input $x(t) = \cos(t) \cdot u(t)$.



Q2:

(i)

$$i_{R_1}(t) = \frac{y(t) - v(t)}{R_1}$$

$$i_{R_2}(t) = \frac{y(t)}{R_2}$$

$$i_c(t) = C\frac{dy(t)}{dt}$$

$$i_{R_1} + i_{R_2} + i_c = 0 \Longrightarrow C\frac{dy(t)}{dt} + \frac{y(t)}{R_2} + \frac{y(t) - v(t)}{R_1} = 0$$

$$\Longrightarrow \frac{dy(t)}{dt} + \frac{R_1 + R_2}{CR_1R_2} y(t) = \frac{1}{CR_1} v(t)$$

iii) $R_1 = 1$, $R2 = 2$, $C = 1$.

$$\Longrightarrow \frac{dy(t)}{dt} + \frac{3}{2} y(t) = v(t)$$

characteristic equation: $s + \frac{3}{2} = 0$

Homogenous component: $S = -\frac{3}{2}$

$$y_h(t) = A \cdot e^{-\frac{3}{2}t}$$

particular component.

$$x(t) = \cos(t)\,u(t) \Longrightarrow y_p(t) = C_1 \sin t + C_2 \cos t$$

This page is for Q2.

ii) The system can be either linear or nonlinear, depending on the initial conditions.

The system is time-invariant.

The system has memory.

The system is causal.

The system is not invertible if the initial conditions are not zero

The system is stable

$$C_1 \cos t - C_2 \sin t + \frac{3}{2}(C_1 \sin t + C_2 \cos t) = \cos t$$

$$\Rightarrow \begin{cases} C_1 + \frac{3}{2} C_2 = 1 \\ \frac{3}{2} C_1 - C_2 = 0 \end{cases} \Rightarrow C_2 = \frac{3}{2} C_1$$

$$\Rightarrow C_1 + \frac{9}{4} C_1 = 1 \Rightarrow C_1 = \frac{4}{13} \quad C_2 = \frac{6}{13}$$

$$y_p(t) = \frac{4}{13} \sin t + \frac{6}{13} \cos t$$

Complete responses:

$$y(t) = A \cdot e^{-\frac{3}{2}t} + \frac{4}{13} \sin t + \frac{6}{13} \cos t.$$

$$y(0) = A + \frac{6}{13} = 0. \Rightarrow A = -\frac{6}{13}$$

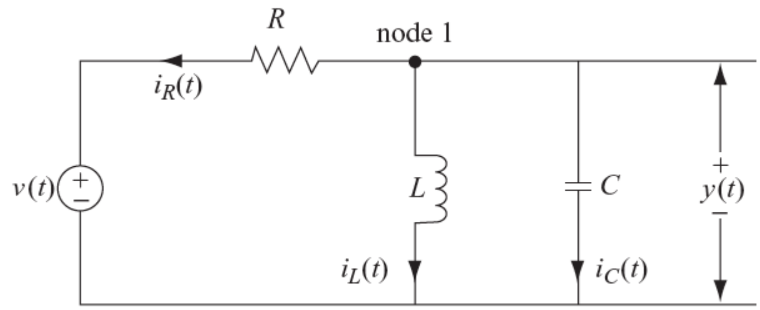
Finally:

$$y(t) = \left[-\frac{6}{13} e^{-\frac{3}{2}t} + \frac{4}{13} \sin t + \frac{6}{13} \cos t \right] u(t)$$

3. (24 marks) Given the following electrical circuit, assume $C = 1,\ R = \frac{1}{2}, L = \frac{1}{2}$,

(i) (8 marks) Determine the differential equation relating the input voltage $v(t)$ to the output voltage $y(t)$.

(ii) (16 marks) Determine the output signal $y(t)$ when we apply the input signal $v(t) = e^{-2t}$ at the time instance $t = 0$, Assume $y(0^-) = 5, y'(0^-) = -2$.



Q3:

(i) $i_R(t) = \dfrac{y(t) - v(t)}{R}$

$i_C(t) = C\,\dfrac{dy(t)}{dt}$

$i_L(t) = \dfrac{1}{L}\int_{-\infty}^{t} y(\tau)\,d\tau$

$i_R + i_L + i_C = 0$

$$\Rightarrow \frac{y(t) - v(t)}{R} + C \cdot \frac{dy(t)}{dt} + \frac{1}{L}\int_{-\infty}^{t} y(\tau)\,d\tau = 0$$

$$\Rightarrow C \cdot \frac{d^2y(t)}{dt^2} + \frac{1}{R} \cdot \frac{dy(t)}{dt} + \frac{1}{L}\,y(t) = \frac{1}{R}\,\frac{dv(t)}{dt}$$

$$\Rightarrow \frac{d^2y(t)}{dt^2} + \frac{1}{RC} \cdot \frac{dy(t)}{dt} + \frac{1}{LC}\,y(t) = \frac{1}{RC}\,\frac{dv(t)}{dt}$$

$C = 1,\ R = \frac{1}{2},\ L = \frac{1}{2}$

$$\frac{d^2y(t)}{dt^2} + 2\,\frac{dy(t)}{dt} + 2\,y(t) = \underbrace{2\,\frac{dv(t)}{dt}}$$

$$x(t) = 2\,\frac{dv(t)}{dt} = -4e^{-2t}$$

ii) homogenous component.

characteric equation.

$$s^2+2s+2=0 \Rightarrow (s+1)^2+1=0$$

$$\Rightarrow s=-1\pm j$$

$$y_h(t) = A_1 e^{-t}\cos t + A_2 e^{-t}\sin t$$

Particalar componet:

$$x(t) = -4e^{-2t} \Rightarrow y(t) = c\,e^{-2t}$$

$$\cancel{4c\cdot e^{-2t}} - \cancel{4ce^{-2t}} + 2c\cdot e^{-2t} = -4e^{-2t}$$

$$c=-2$$

complete response:

$$y(t) = A_1\cdot e^{-t}\cos t + A_2 e^{-t}\sin t - 2e^{-2t}$$

$$y(0) = A_1 - 2 = 5$$

$$y'(t) = -A_1\cdot e^{-t}\sin t - A_1 e^{-t}\cos t + A_2 e^{-t}\cos t - A_2 e^{-t}\sin t + 4e^{-2t}$$

$$y'(0) = -A_1 + A_2 + 4 = -2 \Rightarrow \begin{cases} A_1 = 7 \\ A_2 = 1 \end{cases}$$

4. (10 marks) Prove: $x(t)=\int\limits_{-\infty}^{+\infty} x(\tau)\cdot\delta(t-\tau)\,d\tau$.

I) use the property of impulse signals.

$$
\begin{aligned}
&\int\limits_{-\infty}^{+\infty} x(\tau)\cdot\delta(t-\tau)\,d\tau \\
&=\int\limits_{-\infty}^{+\infty} x(\tau)\big|_{\tau=t}\cdot\delta(t-\tau)\,d\tau \\
&=x(t)\int\limits_{-\infty}^{+\infty}\delta(t-\tau)\,d\tau \\
&=x(t)
\end{aligned}
$$

II) use limit to the rectangular approximation.