

# Concurrent Object-Oriented Languages

## Formal Verification

`wiki.eecs.yorku.ca/course/6490A`

# What is verification?

“Have you made what you were trying to make?”



How the customer explained it



How the Project Leader understood it



How the Analyst designed it



How the Programmer wrote it



How the Business Consultant described it



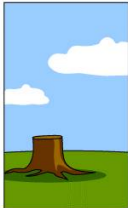
How the project was documented



What operations installed



How the customer was billed



How it was supported



What the customer really needed

Source: Paragon Innovations



# What is verification?

“Have you made what you were trying to make?”

Does the code satisfy all the properties obtained from its specification?



“Have you made the right thing?”

Is the specification of the system correct?

which is also known as *validation*.



# Why do we verify?

Bugs are everywhere.



Source: Bruce Campbell

## 1968 Brazilian Beetle



Source: Dan Palatnik

# Classic bug

“A clear example of the risks of poor programming and verification techniques is the tragic story of the Therac-25—one in a series of radiation therapy machines developed and sold over a number of years by Atomic Energy Canada Limited (AECL). As a direct result of inadequate programming techniques and verification techniques, at least six patients received massive radiation overdoses which caused great pain and suffering and from which three died.”



Source: unknown

# Classic bug

“A computer malfunction at Bank of New York brought the Treasury bond market’s deliveries and payments systems to a near standstill for almost 28 hours ... it seems that the primary error occurred in a messaging system which buffered messages going in and out of the bank. The actual error was an overflow in a counter which was only 16 bits wide, instead of the usual 32. This caused a message database to become corrupted. The programmers and operators, working under tremendous pressure to solve the problem quickly, accidentally copied the corrupt copy of the database over the backup, instead of the other way around.”

Wall Street Journal, November 25, 1985



Source: unknown



# Classic bug

“To correct an anomaly that caused inaccurate results on some high-precision calculations, Intel Corp. last week confirmed that it had updated the floating-point unit (FPU) in the Pentium microprocessor. The company said that the glitch was discovered midyear and was fixed with a mask change in recent silicon. “This was a very rare condition that happened once every 9 to 10 billion operand pairs,” said Steve Smith, a Pentium engineering manager at Intel.”

EE Times, November 7, 1994



Source: Konstantin Lanzet

# Classic bug

“On 4 June 1996, the maiden flight of the Ariane 5 launcher ended in a failure. Only about 40 seconds after initiation of the flight sequence, at an altitude of about 3700 meters, the launcher veered off its flight path, broke up and exploded. . . . The reason why the active SRI 2 did not send correct attitude data was that the unit had declared a failure due to a software exception. . . . The data conversion instructions (in Ada code) were not protected from causing an operand error, although other conversions of comparable variables in the same place in the code were protected.”

Report of the Ariane Inquiry Board



Source: unknown

2012 Beetle



Source: unknown

## The Toronto Skyline



Source: unknown

The Toronto Skyline on August 14, 2003

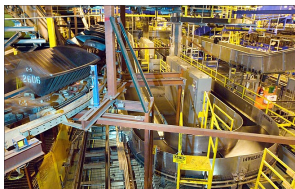


Source: unknown

# Bug of the 21st century

“Ten years ago, the new Denver International Airport marched boldly into the future with a computerized baggage-handling system that immediately became famous for its ability to mangle or misplace a good portion of everything that wandered into its path. Now the book is closing on the brilliant machine that couldn’t sort straight. Sometime over the next few weeks, in an anticlimactic moment marked and mourned by just about nobody, the only airline that ever used any part of the system will pull the plug.”

New York Times, August 27, 2005



Source: Kevin Moloney

# Bug of the 21st century

“When it comes to lethal bugs, the computer glitch that set fire to \$440 million of Knight Capital Group’s funds last Wednesday ranks right up there with the tsetse fly.

In less than an hour, Knight Capital’s computers executed a series of automatic orders that were supposed to be spread out over a period of days. Millions of shares changed hands. The resulting loss, which was nearly four times the company’s 2011 profit, crippled the firm and brought it to the edge of bankruptcy.”

Brian Patrick Eha, CNN, August 9, 2012



Source: CNN

# What's the price tag?

Bank of New York bug: \$ 5 million

Pentium bug: \$ 475 million

Ariane bug: \$ 500 million

Blackout bug: \$ 6 billion

Denver bug: \$ 300 million

Knight bug: \$ 440 million

“The cost of software bugs to the U.S. economy is estimated at \$ 60 billion per year.”

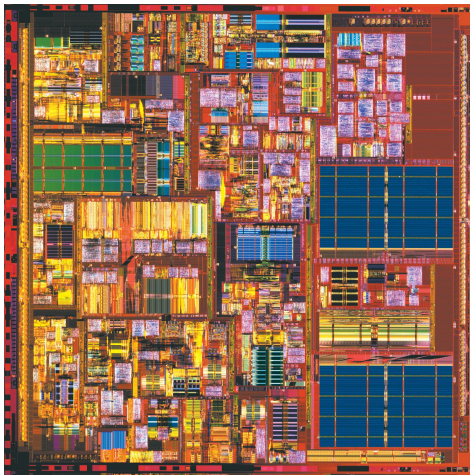
National Institute of Standards and Technology, 2002



# Why are bugs introduced?

Hardware and software systems are among the most complex artifacts ever produced by humans.

# Pentium 4 microprocessor



Source: unknown

Based on slide of Prof. Tom Melham

- transistors:  
55 million
- area:  
146 mm<sup>2</sup>
- clock rate:  
 $3.2 \times 10^9$  Hz

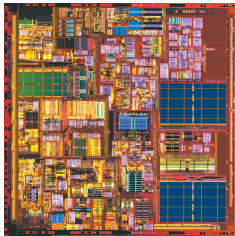
... the connections on a microprocessor were roads in Scotland, ...

Area of microprocessor:  $146 \text{ mm}^2$

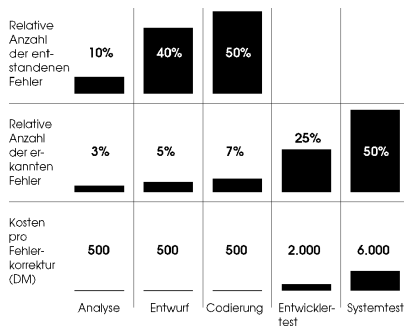
Area of Scotland:  $80,234 \text{ km}^2$

Scale:  $12 \text{ mm} / 283 \text{ km} \approx 1 / 14,150,000$

... then, since each connection is  $0.13 \mu\text{m}$  wide, the roads in Scotland would be 1.84 m wide, 1.84 m apart and eight layers deep!

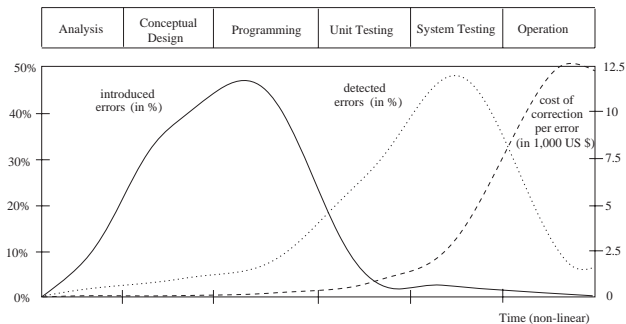


# When are bugs introduced and detected?



Peter Liggesmeyer, Martin Rothfelder, Michael Rettelbach, and Thomas Ackermann. Qualitätssicherung Software-basierter technischer Systeme – Problembereiche und Lösungsansätze. *Informatik-Spektrum*, 21(5):249–258, October 1998.

# When are bugs introduced and detected?



Christel Baier and Joost-Pieter Katoen. *Principles of Model Checking*. The MIT Press. Cambridge, MA, USA. 2008.

# How are bugs detected?

- Manual inspection (very costly and error prone)
- Simulation
- Testing
- Verification

# Limitations of simulation

How long does it take to simulate a 128-bit multiplier on a 3 GHz machine?

Based on slide of Prof. Tom Melham

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How many years is that?

$$2 \times 10^{59}$$

Based on slide of Prof. Tom Melham

# Limitations of testing

“Testing shows the presence, not the absence of bugs.”

Edsger Wybe Dijkstra, October 1969



# How does verification detect bugs?

- Property checking: Does system  $S$  satisfy property  $P$ ?
- Equivalence checking: Are systems  $S_1$  and  $S_2$  equivalent?

We will focus on *property checking*.

# What is formal about formal verification?

- The system is *formalized* (as some mathematical entity).
- The property is *formalized* in some logic.
- Whether the system satisfies the property is *formalized* as well.

$$(n + 1)^2 = n^2 + 2n + 1?$$

## Simulation

$n$	$(n + 1)^2$	$n^2 + 2n + 1$
0	1	1
1	4	4
2	9	9
3	16	16
4	25	25
...	...	...

Based on slide of Prof. Tom Melham

$$(n + 1)^2 = n^2 + 2n + 1?$$

## Testing

$n$	$(n + 1)^2$	$n^2 + 2n + 1$
0	1	1
7	64	64
34	1156	1156
78	6241	6241
101	10404	10404
...	...	...

Based on slide of Prof. Tom Melham

$$(n + 1)^2 = n^2 + 2n + 1?$$

## Formal verification

$$\begin{aligned}(n + 1)^2 &= (n + 1) \times (n + 1) \\ &= (n \times n) + (n \times 1) + (1 \times n) + (1 \times 1) \\ &= n^2 + n + n + 1 \\ &= n^2 + 2n + 1\end{aligned}$$

Based on slide of Prof. Tom Melham

# Why is formal verification computer-aided?

- A formal proof that a system satisfies a property can consist of millions of steps.
- Without the use of clever algorithms and data structures, proving that a system satisfies a property can be totally impractical.

# Model of a system

A **model** of a system is an **abstraction** of the system.



There are many levels of abstraction and, hence, a system can be modelled in many different ways.

## Definition

A **transition system** is a tuple  $\langle S, \rightarrow, I, AP, L \rangle$  consisting of

- a set  $S$  of **states**,
- a **transition relation**  $\rightarrow \subseteq S \times S$ ,
- a set  $I \subseteq S$  of **initial states**,
- a set  $AP$  of **atomic propositions**, and
- a **labelling function**  $L : S \rightarrow 2^{AP}$ .



# Model of a traffic light

A traffic light can be red, green, amber or black (not working). The traffic light might stop working at any time. After it has been repaired, it turns red. Initially, the light is red.

How many different states do we need to model the traffic light?

# Model of a traffic light

A traffic light can be red, green, amber or black (not working). The traffic light might stop working at any time. After it has been repaired, it turns red. Initially, the light is red.

$$S = \{1, 2, 3, 4\}$$

- 1 red
- 2 green
- 3 amber
- 4 black

What are the transitions?

# Model of a traffic light

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$$\begin{aligned} S &= \{1, 2, 3, 4, 5\} \\ \rightarrow &= \{(1, 2), (2, 3), (3, 1), (1, 4), (2, 4), (3, 4), (4, 1)\} \end{aligned}$$

Which are the initial states?

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$$\begin{aligned}S &= \{1, 2, 3, 4, 5\} \\ \rightarrow &= \{(1, 2), (2, 3), (3, 1), (1, 4), (2, 4), (3, 4), (4, 1)\} \\ I &= \{1\} \\ AP &= \{r, a, g\}\end{aligned}$$

How is the labelling function defined?

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$$\begin{aligned} S &= \{1, 2, 3, 4, 5\} \\ \rightarrow &= \{(1, 2), (2, 3), (3, 1), (1, 4), (2, 4), (3, 4), (4, 1)\} \\ I &= \{1\} \\ AP &= \{r, a, g\} \\ L &= \{1 \mapsto \{r\}, 2 \mapsto \{r, a\}, 3 \mapsto \{g\}, 4 \mapsto \{a\}, 5 \mapsto \emptyset\} \end{aligned}$$

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