

1.

$$x(t) = A \cos(\omega_0 t + \theta) = \frac{A}{2} e^{j(\omega_0 t + \theta)} + \frac{A}{2} e^{-j(\omega_0 t + \theta)}$$

$$= \frac{A}{2} e^{j\theta} e^{j\omega_0 t} + \frac{A}{2} e^{-j\theta} e^{-j\omega_0 t}$$

using  $\frac{x(f)}{e^{j2\pi f t}} \quad X(j\omega) \quad X(f)$   
 $2\pi\delta(\omega - \omega_0) \quad \delta(f - f_0)$

$$X(j\omega) = \frac{A}{2} e^{j\theta} 2\pi\delta(\omega - \omega_0) + \frac{A}{2} e^{-j\theta} 2\pi\delta(\omega + \omega_0)$$

$$= A\pi e^{j\theta} \delta(\omega - \omega_0) + A\pi e^{-j\theta} \delta(\omega + \omega_0)$$

$$X(f) = \frac{A}{2} e^{j\theta} \delta(f - f_0) + \frac{A}{2} e^{-j\theta} \delta(f + f_0)$$

-or-  
 invoke F.S. expression

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

clearly  $c_1 = \frac{A}{2} e^{j\theta}$  and  $c_{-1} = \frac{A}{2} e^{-j\theta}$

then do F.T. as discussed in Lab 1.

2.

$$x(t) = j5 e^{-j2\pi 1000t} + 3e^{j\frac{\pi}{6}} e^{-j2\pi 60t} + 16 + 3e^{-j\frac{\pi}{6}} e^{j2\pi 60t} - j5 e^{j2\pi 1000t}$$

$$= 16 + 3 \left( e^{-j(2\pi 60t - \frac{\pi}{6})} + e^{j(2\pi 60t - \frac{\pi}{6})} \right) + j5 \left( e^{-j2\pi 1000t} - e^{j2\pi 1000t} \right)$$

$$= 16 + 6 \cos\left(2\pi 60t - \frac{\pi}{6}\right) - 10 \sin(2\pi 1000t)$$

3.

a) power signal (periodic)  $P_x = \frac{A^2}{2}$  (avg. power over period)

b) energy signal (finite in time)  $E_x = \frac{A^2 T_0}{2}$

c) energy " (finite in time)

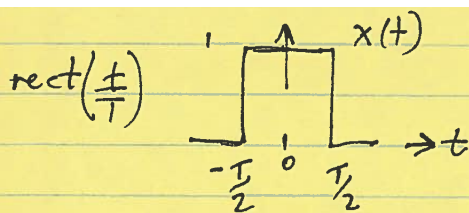
$$E_x = \int_0^{\infty} A^2 e^{-2at} dt = \frac{A^2}{2a}$$

d) power signal (periodic)

fundamental freq  $f_0 = \frac{1}{2\pi}$   $T_0 = 2\pi$

$$P_x = 13$$

4.



$$E_x = \int_{-\infty}^{\infty} x^2(t) dt = T \leftarrow \text{normalized energy}$$

5.

we stated in class (from Parseval's thm. for F.S.)

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0) \quad P_x = \int_{-\infty}^{\infty} G_x(f) df = \sum_{n=-\infty}^{\infty} |c_n|^2$$

(from sampling property of Dirac  $\delta$  fn.)

6.

for periodic fn.

$$R_x(\tau) = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} [A \cos(2\pi f_0 t + \phi) A \cos(2\pi f_0 t + 2\pi f_0 \tau + \phi)] dt$$

$$K = 2\pi f_0 t + \phi$$

$$L = 2\pi f_0 \tau$$

$$\begin{aligned} & \cos(K) \cos(K+L) \\ & \cos(K) [\cos(K) \cos(L) - \sin(K) \sin(L)] \\ & \cos^2(K) \cos(L) - \sin(K) \sin(L) \cos(K) \\ & \left( \frac{1 + \cos 2K}{2} \right) \cos(L) - \underbrace{\sin(K) \sin(L) \cos(K)} \end{aligned}$$

$\uparrow$  integrates to zero over  $T_0$       integrates to 0 over  $T_0$

$$R_x(\tau) = \frac{A^2}{2} \cos(2\pi f_0 \tau)$$

$$P_x = R_x(0) = \frac{A^2}{2}$$

7.

a) avg. val of  $x(t) = 1 + \cos 2\pi f_0 t$

$$\langle x(t) \rangle = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} (1 + \cos 2\pi f_0 t) dt = 1 = \mu$$

b) ac power

$$\sigma^2(t) = \langle (x - \mu)^2 \rangle = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos^2 2\pi f_0 t dt = \frac{1}{2}$$

c) rms val of  $x(t)$

$$\left[ \langle x^2(t) \rangle \right]^{\frac{1}{2}} = \left[ \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} (1 + \cos 2\pi f_0 t)^2 dt \right]^{\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

8.

$$X(f) = \text{sinc } f = \frac{\sin \pi f}{\pi f}$$

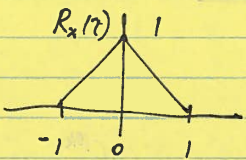
$$\psi_x(f) = |X(f)|^2 = \text{sinc}^2(f)$$

$$|X(f)|^2 = \text{F.T.} \{ R_x(\tau) \}$$

→ looking up Table A.1 of Sklar...

$$\text{F.T.} \left\{ 1 - \frac{|\tau|}{T} \right\} = T \text{sinc}^2(fT)$$

∴ in our case  $R_x(\tau) = \begin{cases} 1 - |\tau| & |\tau| < 1 \\ 0 & \text{otherwise} \end{cases}$





9.

$$\int_{-\infty}^{\infty} e^{-t^2} \delta(t-2) dt = e^{-2^2} = 0.0183$$

10.

$$X_2(f) = k [\delta(f-f_0) + \delta(f+f_0)]$$

$$X_1 * X_2 = X_1 * k [\delta(f-f_0) + \delta(f+f_0)]$$

$$C(f) = \int_{-\infty}^{\infty} X_1(f-s) k [\delta(s-f_0) + \delta(s+f_0)] ds$$

$$= k \cdot [X_1(f-f_0) + X_1(f+f_0)]$$

$$k \cdot \left[ \begin{array}{c} \text{triangle at } 0 \\ \text{triangle at } -2f_0 \end{array} + \begin{array}{c} \text{triangle at } 0 \\ \text{triangle at } 2f_0 \end{array} \right]$$

$$= \begin{array}{c} \text{triangle at } 0 \text{ with height } 2k \\ \text{triangle at } -2f_0 \text{ with height } k \\ \text{triangle at } 2f_0 \text{ with height } k \end{array}$$

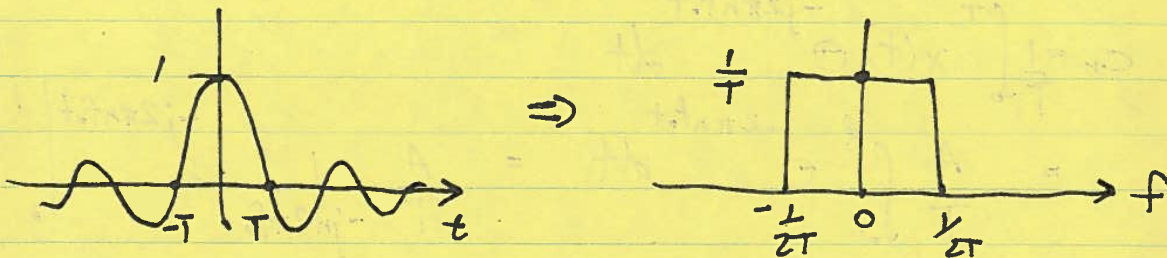
11.

$$\begin{aligned}
 a) \quad P_x &= \int_{-10k}^{10k} G_x df = 2 \int_0^{10k} G_x df = 2 \int_0^{10k} 10^{-6} f^2 df \\
 &= 2 \left[ \frac{10^{-6} f^3}{3} \right] \Big|_0^{10k} = 667 \text{ kW}
 \end{aligned}$$

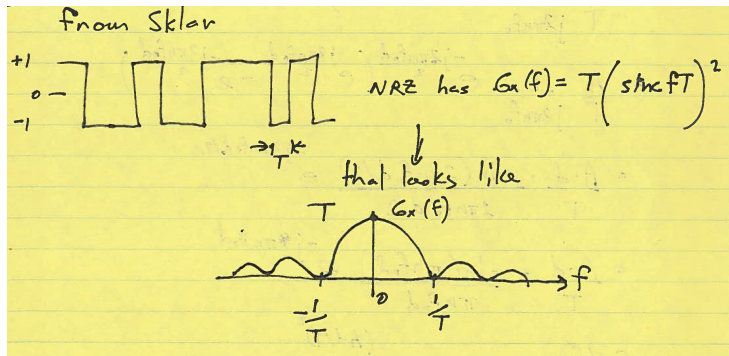
$$b) \quad 2 \int_{5k}^{6k} 10^{-6} f^2 = 2 \left[ \frac{10^{-6} f^3}{3} \right] \Big|_{5k}^{6k} = 60.7 \text{ kW}$$

12.

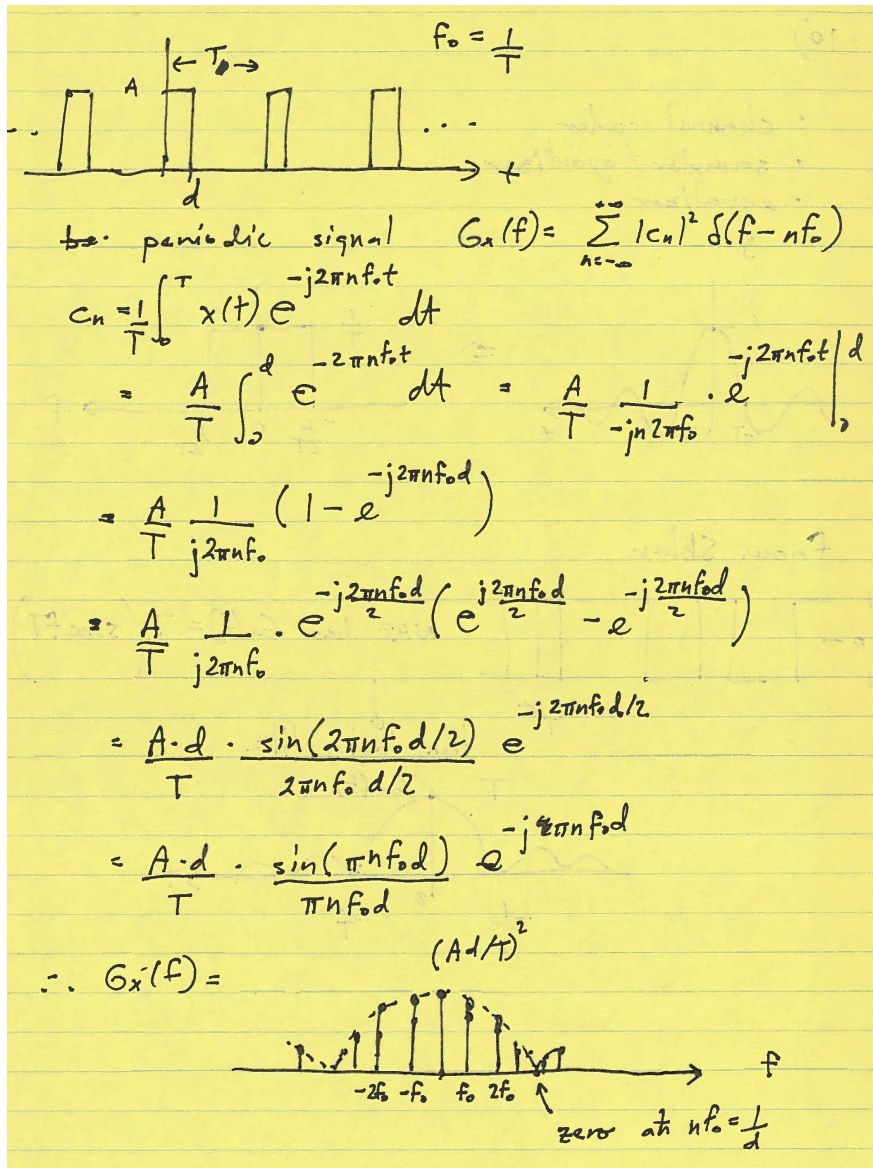
- channel coder
- sampler / quantizer
- equalizer
- )



13.



14.



15.

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \\ &= 2 + e^{-j\omega} + e^{-j3\omega} \end{aligned}$$