1. 1.20 (a)-(d)

a) $\frac{1}{2}$ power BW

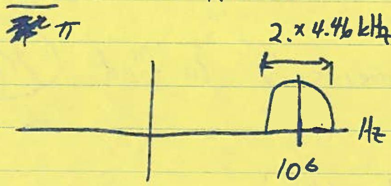
→ at what f does $G_x(f)$ drop to $\frac{1}{2}$ its pk. val.

$$\frac{1}{2} = \left[\frac{\sin(\pi f_0 \times 10^{-4})}{\pi f_0 \, 10^{-4}} \right]^{\frac{1}{2}} \quad \text{(forget about offset)}$$

$$\frac{\sin \pi x}{\pi x} = \text{sinc}(x) = \frac{1}{\sqrt{2}} \quad \text{at } x \simeq \frac{1.4}{\pi}$$

$\therefore$ $f_0 \times 10^{-4} = \frac{1.4}{\pi}$ represents $\frac{1}{2}$ power point

$$f_0 = \frac{1.4 \times 10^{4}}{\pi} = 4.46\,kHz$$



$BW \simeq 2 \times 4.46 \simeq 9\,kHz$

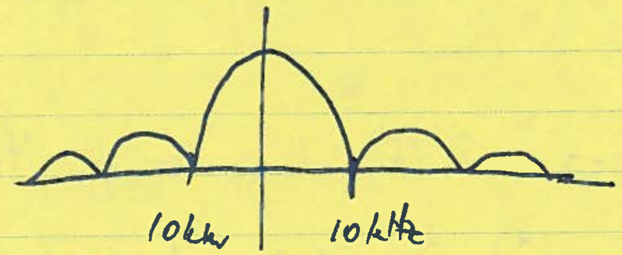
b) noise equiv BW :

$$W_N \, G_x(f_c) = P_x$$

$$W_N \cdot 1 = 2\int_0^{\infty} \left[\frac{\sin(\pi f 10^{-4})}{\pi f 10^{-4}} \right]^2 df = 2\times 10^4 \overbrace{\int_0^{\infty} \left(\frac{\sin \pi x}{\pi x} \right)^2 dx}^{1/2}$$

$x = f \times 10^{-4}$ $\qquad = 10\,kHz$

c) null-to-null BW



$= 20\,kHz$

d) 99% pwr. BW

$$0.99 = \frac{10^{-4}\int_0^{f_0} \mathrm{sinc}^2(f \times 10^{-4})\,df}{10^{-4}\int_0^{\infty} \mathrm{sinc}^2(f \times 10^{-4})\,df}$$

solve numerically to find 206 kHz

2. 2.2

a) 6400 bps b) 1600 symbols/s $= \frac{6400\text{ bps}}{4\text{ bits/symbol}}$

3. 2.4

$$x_s(t) = x(t) \cdot p(t)$$

$$= x(t)\left\{\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s t}\right\}$$

j·sin's cancel

$$= x(t) \cdot \left\{ c_0 + 2\sum_{n=1}^{\infty} c_n \cos 2\pi n f_s t \right\}$$

$$x_1(t) = x_s(t) \cos 2\pi m f_s t \qquad x_s \to \otimes \to x_1(t)$$

$$= x(t) \cdot \Big(\underbrace{c_0 \cos 2\pi m f_s t + 2\sum_{\substack{n=1 \\ n \neq m}}^{\infty} c_n \cos 2\pi n f_s t \cos 2\pi m f_s t}_{\text{removed by LPF}}$$

$$+ 2 c_m \cos^2 2\pi m f_s t \Big)$$

$$x_1(t) \to \boxed{LPF} \to x_0(t) = x(t) \cdot 2 c_m \left(\frac{1}{2} + \underbrace{\frac{1}{2} \cos 4\pi m f_s t}_{\text{also removed by LPF}} \right)$$

$$= c_m x(t)$$

4. 2.5

a) $b \geq \log_2 L$

b: # of bits needed for L levels

$\therefore$ b ~~bits~~ bits need to be sent at least every T_s

$R \geq b \cdot f_s$

$R \geq \log_2 L \cdot \frac{1}{T_s}$

$\frac{1}{R} = T \leq \frac{T_s}{\log_2 L}$

b) equality is valid if L is a power of 2

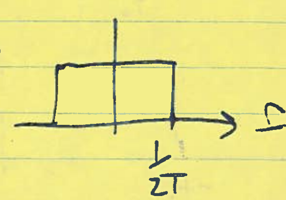
5. 2.6

a) 32 b) 256 c) 2^x

6. 2.7

$x(t) = \frac{\sin 6280t}{6280t} = \frac{\sin \pi t/T}{\pi t/T}$

$\therefore T = 5 \times 10^{-4}$ s $\Rightarrow$



$f_m = \frac{1}{10 \times 10^{-4}} = 1$ kHz

$\therefore f_s = 2000$ Samples/second

7. 2.8

a) $SNR_q = 3L^2$ (eqn. 2.20 in text)

$10 \log(3L^2) \geq 30\,dB$

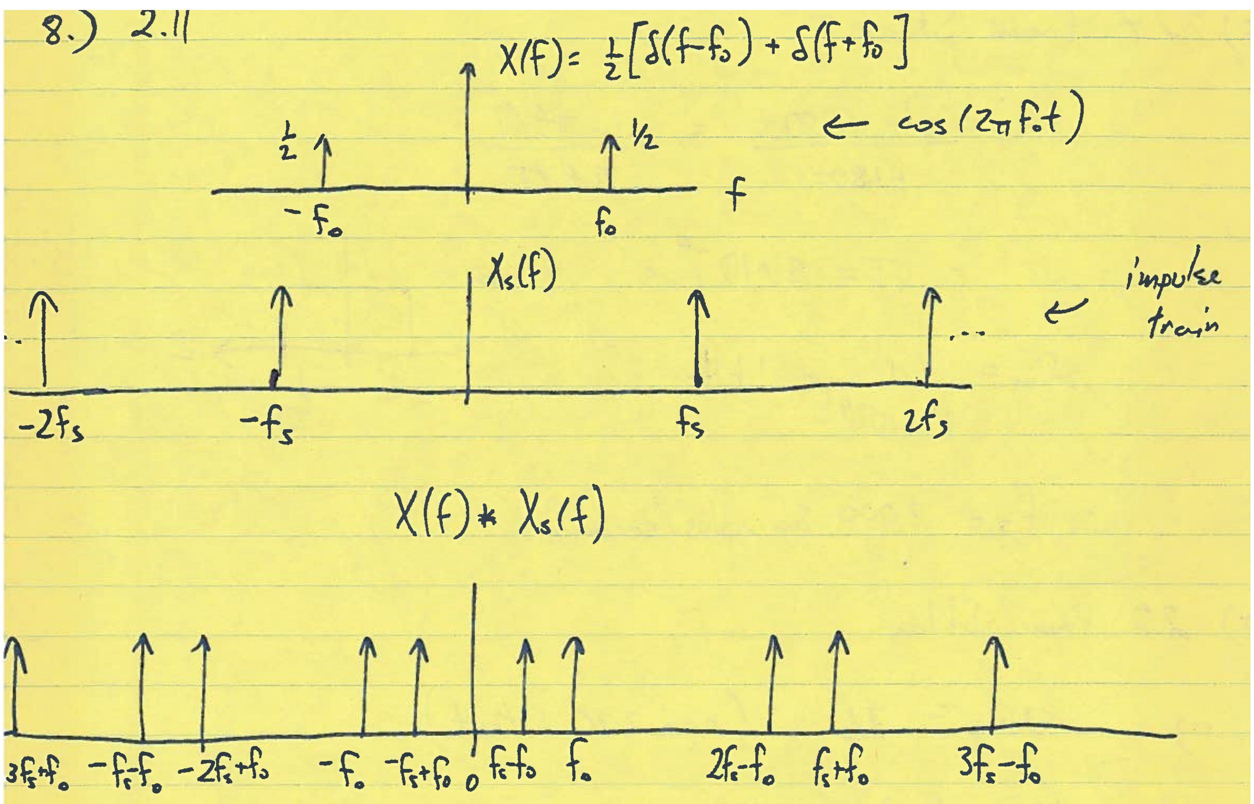
$L = \lceil 18.26 \rceil = 19$ } min number of quantization levels

$\ell = \lceil \log_2 L \rceil = 5$ bits/sample

b) $T_b = \frac{T_s}{\ell} = \frac{1}{\ell f_s} = \frac{1}{5 \times 8k} = 25\mu s$

$W = \frac{1}{T_b} = \frac{1}{25\mu s} = 400\,kHz$ } 1st lobe of rect pulse

8. 2.11



9. 2.15

Binary case:

$R = 8000 \text{ samples/s} \times 6 \text{ bits/sample} = 48 \text{ kbps}$

$W = \dfrac{1}{T_b} = R = 48 \text{ kHz}$

$SNR_q = 3L^2 = 3(64)^2 = 12{,}288 \simeq 41 \text{dB}$

4-level case:

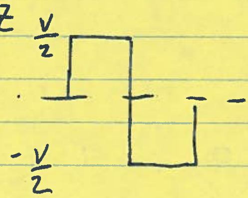
$R_s = \dfrac{48 \text{ kbps}}{2 \text{ bits/symbol}} = 24k \text{ symbols/s}$

$W = \dfrac{1}{T} = R_s = 24 \text{ kHz}$

$SNR_q = 41 \text{ dB}$

10. 2.17

bipolar NRZ



$= \dfrac{V^2}{4} = \dfrac{1}{2}\left(\dfrac{V}{2}\right)^2 + \dfrac{1}{2}\left(\dfrac{V}{2}\right)^2$

unipolar NRZ

$\dfrac{V^2}{2} = \dfrac{1}{2}\left(V^2\right) + \dfrac{1}{2}(0)^2$

- bipolar needs half the power
- more sophisticated design needed to produce symmetrical wave form

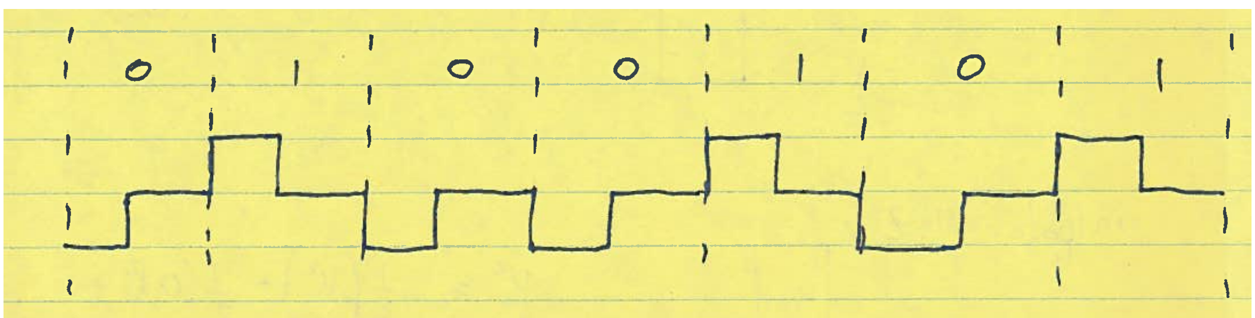
11. 2.18

$$\text{T1 data rate} = 24\,\frac{\text{samples}}{\text{frame}} \times 8\,\frac{\text{bits}}{\text{sample}} \times 8000\,\frac{\text{frames}}{\text{s}} + 1\,\frac{\text{bit align}}{\text{frame}}$$

$$= 193\,\frac{\text{bits}}{\text{frame}} \times 8000\,\frac{\text{frames}}{\text{s}} = 1.544\times10^{6}\ \text{bps}$$

$$\text{bandwidth efficiency}\ \eta = \frac{R}{W} = \frac{1.544\times10^{6}}{386\times10^{3}} = 4\,\frac{\text{bps}}{\text{Hz}}$$

12. Bipolar RZ



13. Gaussian RV

$$1 - 2Q\left(\frac{6}{3}\right) = 1 - 2Q(2) = 1 - 2\times0.0228 = 0.9545$$

14. Uniform random variable

$$p_x(x) = \begin{cases} k & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow$ probability theory requires

$$1 = \int_{-\infty}^{\infty} p_x(x)\,dx = \int_a^b k\,dx = k(b-a) = 1$$

$$k = \frac{1}{b-a}$$

a for $a=-1$, $b=2$ $\quad k = \frac{1}{3}$

$$P(|X| \le \tfrac{1}{2}) = P(-\tfrac{1}{2} \le X \le \tfrac{1}{2}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} p_x(x)\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \tfrac{1}{3}\,dx = \boxed{\tfrac{1}{3}}$$

15. Binary block with errors

- just like flipping coin

recall is $\{H\}=1$ & $\{T\}=0$

$$p(X=1) = p$$

$$p(X=0) = 1-p$$

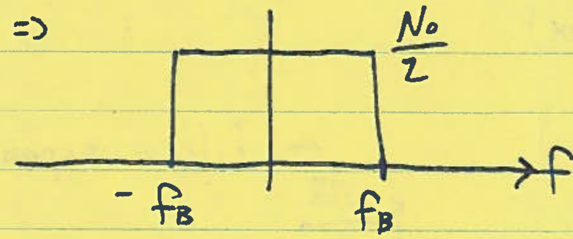
- mean $= p$ $\qquad$ variance $= p(1-p)$
- now p is probability of error
- and we flip the coin $n=16$ times so

$$\mu = n \cdot p = 16 \times 0.01 = 0.16$$

$$\sigma_x^2 = n \cdot p \cdot (1-p) = 16 \times 0.01 \times 0.99 = 0.158$$

16. Filtered white noise

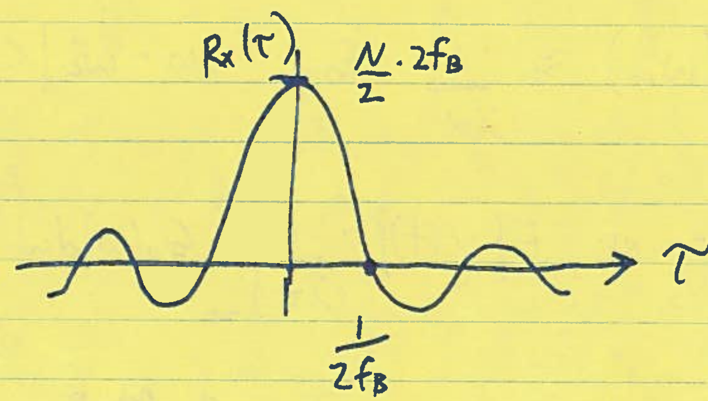
$G_x(f) \Rightarrow$



$$R_x(\tau) = F.T.^{-1}\{G_x(f)\}$$

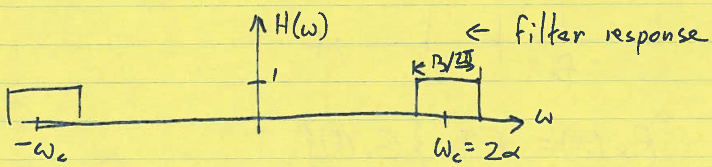
$$R_x(\tau) = \int_{-f_B}^{f_B} \frac{N}{2} e^{j2\pi f\tau} df$$

$$= \frac{N}{2} \cdot 2f_B \cdot \frac{\sin(2\pi f_B \tau)}{2\pi f_B \tau}$$



17. BPF'd noise

$$G_x(\omega) = A^2 \cdot \frac{4\alpha}{\omega^2 + (2\alpha)^2}$$



$$G_Y(\omega) = |H(\omega)|^2 G_x(\omega)$$

since $H(0) = 0$ clearly DC component $= 0$

at ω_c $G_x(\omega_c) = A^2 \cdot 4\alpha / 2(2\alpha)^2 = A^2/2\alpha$

around $\omega_c >> B/2\pi$

filter output $\Rightarrow G_y(\omega_c) \approx \frac{A^2}{2\alpha}$ for $|\omega - \omega_c| < \frac{B}{4\pi}$

avg. output power $E\{y^2(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_y(\omega)\,d\omega$

$$= \frac{1}{2\pi} \cdot 2 \cdot \frac{B}{2\pi} \cdot \frac{A^2}{2\alpha}$$

$$= \frac{A^2 \cdot B}{(2\pi)^2 \alpha}$$

18. 3.4

$$P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\frac{1-(-1)}{2}\right)$$

$$= Q(1) = 0.1587$$

19. 3.5

$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ $E_b = A^2 T$ for bipolar signalling

& $A=1$ $\therefore$ $E_b = T$

$P_B = Q(x) \leq 10^{-3}$

$x = \sqrt{\frac{2E_b}{N_0}} = 3.09$

$\frac{E_b}{N_0} = 4.77$; $\frac{N_0}{2} = 10^{-3}$ (given)

$E_b = T = 4.77 \times 10^{-3} \times 2$

$\therefore$ $R = \frac{1}{T} \leq 104.8$ bps

20. 3.6

a) $P(s_1) = P(s_2) = 0.5$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$\int_0^T dt$ $\int_0^T -1\,dt$

for $P(s_1) = P(s_2)$ $\gamma_0 = \frac{a_1 + a_2}{2} = \frac{T + (-T)}{2} = 0$

b) $P(s_1) = 0.7 \Rightarrow P(s_2) = 0.3$

using B.12

$$\frac{z(a_1 - a_2)}{\sigma_0^2} \underset{H_2}{\overset{H_1}{\gtrless}} \ln \frac{P(s_2)}{P(s_1)}$$

$$z \underset{H_2}{\overset{H_1}{\gtrless}} \frac{\sigma_0^2}{a_1 - a_2} \ln \frac{P(s_2)}{P(s_1)} = \gamma_0$$

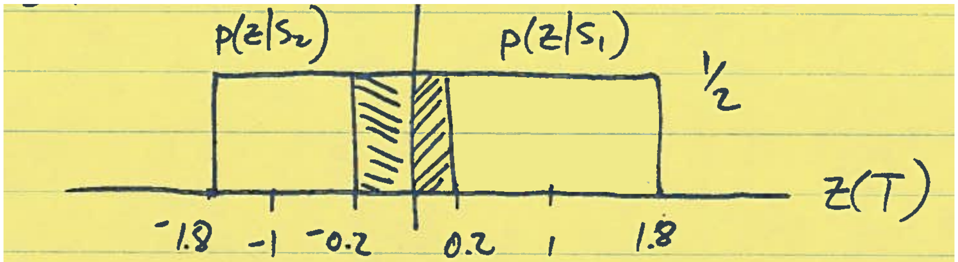
$$\gamma_0 = \frac{0.1}{2T} \ln\left(\frac{0.3}{0.7}\right)$$

$$= \frac{-0.04}{T}$$

c) $\gamma_0 = \frac{0.1}{2T} \ln\left(\frac{0.8}{0.2}\right) = \frac{0.07}{T}$

d) ~~a prior~~ a priori probabilities pull the decision threshold to the more likely possibility (to ~~take place~~ happen in the first place)

21. 3.7



$$P_E = P(s_1)\int_{-0.2}^{0} \frac{1}{2}\,dz + P(s_2)\int_{0}^{0.2} \frac{1}{2}\,dz$$

$$= \left[\frac{1}{2}z\right]_{-0.2}^{0} = \frac{0.2}{2} = 0.1$$

22. 3.8

a) 16 levels $= M = 2^k$ $\quad k = 4$ bits/symbol

$$R_s = \frac{R}{\log_2 M} = \frac{10\ \text{Mbps}}{4\ \text{b/sym}} = 2.5\ \text{M symbols/s}$$

$$\text{min BW} = R_s/2 = 1.25\ \text{MHz}$$

c) Using eqn. 3.80 $\quad W = \frac{1}{2}(1+r)R_s$

$$1.375\ \text{MHz} = (1+r)\cdot 1.25\ \text{MHz}$$

$$r = 0.1$$