> Quiz 3 will be inspired by these questions as well as questions from the midterm, but will probably not be exactly like any one of them or be worded in the same way. It may also ask you questions that build on or apply the ideas explored in the questions below. So, it is important that you understand the underlying ideas, not just be able to parrot them.

1. A communications systems at any time sends any one of three possible messages $\{a, b, c\}$ and receives any one of three related signals $\{1, 2, 3\}$. Ideally if a is sent 1 should be detected, if b is sent 2 should be detected and if c is sent 3 should be detected. Unfortunately our channel is noisy and the likelihood relationship between these inputs and outputs is shown in Fig. 1. For example, from this graph you know that $P(2|a) = 0.3$ and $P(3|c) = 0.8$. Assume that you know $P(a) = 0.3$, $P(b) = 0.5$, $P(c) = 0.2$.

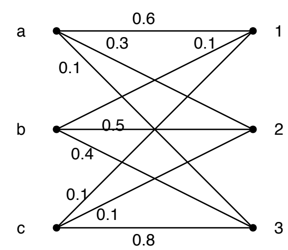

Figure 1: Detector input/output likelihood relations.

Given that you received the symbol 1 what is the decision of the optimum detector on which symbol was transmitted?

Given that the received symbol is 2 and the receiver decides that message b was send, what is the probability that the decision is correct? (i.e. that message b was actually transmitted).

What is the probability of error of the optimum detector?

2. A weird binary communicator represents message m_0 with $s_0 = 0$ and message m_1 with a random variable s with an exponential probability distribution $p_S(s) = a \cdot \exp[-a \cdot s]$ for $s \geq 0$ (and 0 for $s < 0$). The signals, s_0 and s_1 are subject to additive noise that also has an exponential probability distribution, $p_N(n) = b \cdot \exp[-b \cdot b]$ for $n \geq 0$ (and 0 for $n < 0$). Find the decision rule that minimizes the error probability if m_0 and m_1 are equally likely. It may not hurt for you to know that the PDF of the sum of two random variables is equal to the convolution of the individual PDFs.

3. Question 3.12 from the Sklar text.

4. Question 3.13 from the Sklar text.

5. Question 3.14 from the Sklar text.

6. Question 3.15 from the Sklar text.

7. Question 3.17 from the Sklar text.

8. Question 3.18 from the Sklar text.

9. Find the output of the matched filter if the input is a rectangular pulse of height A and duration T. What is the SNR assuming white noise of PSD $N_o/2$.

10. Repeat the problem above assuming an RC filter is used instead of a matched filter.

11. A channel has a discrete-time impulse response of $h[k] = [1, -0.5, 0.1]$. The input signal is $m_k \in \{0, 1\}$ and that the output signal is s_k.

a. Draw the state-transition diagram equivalent of this channel (circles containing the state of the channel, transitions between states labeled with (m_k, s_k).

b. For the question above sketch one stage of the trellis complete trellis diagram (i.e. don't assume any initial or final conditions). Make sure the states are clearly labelled and again demarcate the branches with (m_k, s_k).

c. Draw the trellis assuming your first two output signals are 0.5 and -0.2 (assuming that you started in the all zeros state). Label the branch weights.