1.

- Given that 1 is received... optimum receiver chooses greatest of...

$$P(a|1),\ P(b|1),\ P(c|1)$$

$$P(a|1) = \frac{P(1|a)P(a)}{P(1)}$$

$$P(1) = P(1|a)P(a) + P(1|b)P(b) + P(1|c)P(c) = 0.25$$

$\therefore P(a|1) = 0.72$

similarly

$P(b|1) = 0.2$

$P(c|1) = 0.08$

$\therefore$ opt. detector will decide that a was sent

- if b was sent and 2 is received

$$P(\text{correct decision}|2) = P(b|2) = \frac{P(2|b)P(b)}{P(2)}$$

$= 0.6944$ (you should be able to reach this calculation)

- $P(\text{error}) = 1 - P(\text{correct})$

$$P(\text{correct}) = P(\text{correct}|1)P(1) + P(\text{correct}|2)P(2) + P(\text{corr.}|3)P(3)$$

$P(\text{correct}|1) = P(a|1) = 0.72$

$P(\text{correct}|2) = P(b|2) = 0.6944$

$P(\text{correct}|3) = P(c|3) = 0.5128$

$P(\text{error}) = 0.3727$

2.

$H_0: \quad m_0 \rightarrow r = n$

$H_1: \quad m_1 \rightarrow r = s + n$

$p_s(s) = a e^{-as} \qquad p_N(n) = b e^{-bn}$

$$p_r(m_1 | r) \underset{m_0}{\overset{m_1}{\gtrless}} p_r(m_0 | r)$$

$$p_r(r | m_1) \underset{m_0}{\overset{m_1}{\gtrless}} p_r(r | m_0)$$ ← ML since m_0 + m_1 equally likely

$$p_r(r|m_1) = p_s(r) * p_n(r)$$

$$= \int_0^r p_s(r-\tau) p_s(\tau) \, d\tau$$

$$= \int_0^r a e^{-a(r-\tau)} b e^{-b\tau} \, d\tau$$

$$= \frac{ab}{a-b} \left[e^{-br} - e^{-ar} \right]$$

$$p_r(r|m_0) = p_N(r) = b e^{-br}$$

$$\frac{a}{a-b} \left[1 - e^{(b-a)r} \right] \underset{m_0}{\overset{m_1}{\gtrless}} 1$$

$$r \underset{m_1}{\overset{m_0}{\gtrless}} \frac{1}{b-a} \ln\left[\frac{b}{a}\right]$$

3. 3.12

a) channel BW is 100 kHz = W

- normally $R = 2W$
- with raised cosine employing roll-off factor $r = 0.6$ we have

$$R = \frac{2W}{(1+r)} = \frac{200}{1.6} = 125 \text{ ksymbols/sec.}$$

- since binary ~~wavefo~~ waveforms are used,

$$R_b = R = 125 \text{ kbps}$$

b) $L = 32 = 2^b \quad b = 5$

$\therefore \frac{125}{5} = 25$ k samples/second can be sent $= f_s$

$$\therefore f_{max} = \frac{f_s}{2} = 12.5 \text{ kHz}$$

c) 8-ary PAM $\Rightarrow$ 3 bits per symbol

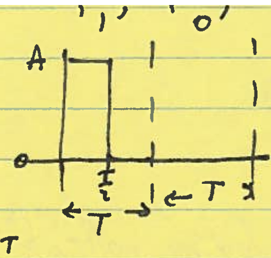
$$\therefore R_b = 3R = 375 \text{ kbps}$$

$$f_s = \frac{375}{5} = 75 \text{ kbps}$$

$$f_{max} = \frac{f_s}{2} = 37.5 \text{ kHz}$$

4. 3.13

RZ pulses . . .



$$E_d = \int_0^T (s_1 - s_2)^2 dt = \frac{A^2 \cdot T}{2}$$

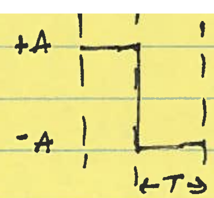
$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 \cdot T}{4N_0}}\right)$$

$$10^{-3} = Q(x) \quad \Rightarrow \quad x = 3.1$$

$$\frac{(0.1)^2 \cdot T}{4 \times 10^{-8}} = (3.1)^2, \; T = 38.4 \mu s \quad \therefore R = \frac{1}{T} = 26 \text{ kbps}$$

5. 3.14

NRZ



$$P_B = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) = 10^{-3} = Q(3.1)$$

$$\therefore \; 3.1 = \sqrt{\frac{2A^2(1/56k)}{10^{-6}}} \qquad A^2 = 0.268$$

$\therefore$ with no signal loss ~ 268 mW are needed

↙ ×2

with 3-dB loss 538 mW are needed

6. 3.15

Nyquist min. bw is $\frac{1}{2T}$ where T is the symbol period (implicit from Nyquist pulse shaping criterion)

⇒ PSD of random bipolar sequence $= T\left(\frac{\sin(\pi f T)}{\pi f T}\right)^2$

P_x = total area $= \int_{-\infty}^{\infty} T\left(\frac{\sin(\pi f T)}{\pi f T}\right)^2 df = ?$

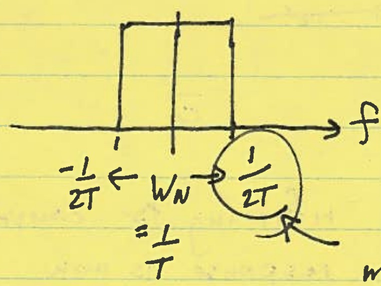
$\int_0^{\infty} \frac{\sin^2 px}{x^2} dx = \frac{\pi p}{2}$

∴ our integral of interest is $= 2 \cdot T \cdot \frac{\pi}{2} \cdot \pi T \cdot \frac{1}{\pi^2 T^2}$

$P_x = 1$!

⇒ ∴ $W_N = \frac{P_x}{G_x(f)|_{pk}} = \frac{1}{T}$ (equiv. noise BW... double sided)

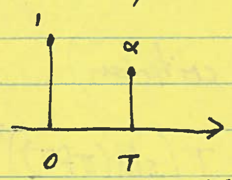
see Sklar pg. 48



matches Nyquist min. BW

7. 3.17

→ overall system impulse response is



$$h(t) = \delta(t) + \alpha\delta(t-T)$$

- the bigger (i.e. more taps) you make your ZFE (zero-forcing equalizer) the "better" (closer to single $\delta(t)$) will your final impulse response be

- e.g. imagine a 4 tap ZFE

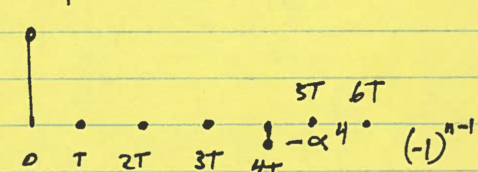
$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ 0 & \alpha & 1 & 0 \\ 0 & 0 & \alpha & 1 \\ \not 0 & \not 0 & \not 0 & \not\alpha \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

← convolution matrix

$$\bar{z} = \bar{X} \quad \bar{c}$$

$c_0 = 1$
$c_1 = -\alpha \cdot c_0 = -\alpha$
$c_2 = -\alpha \cdot c_1 = +\alpha^2$
$c_3 = -\alpha \cdot c_2 = -\alpha^3$

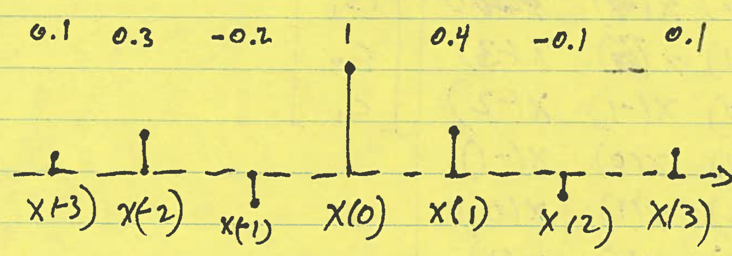
referring to convolution matrix response is now



clearly n-tap ZFE results in net impulse response of $\delta(t) + \delta(t-nT)\alpha^n$

$$h_{total} = \delta(t) + (-1)^{n-1}\delta(t-nT)\cdot\alpha^n$$

8. 3.18



$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

inverting the convolution matrix

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0.8752 & 0.2593 & -0.2107 \\ -0.3079 & 0.8347 & 0.2593 \\ 0.2107 & -0.3079 & 0.8752 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$c_{-1} = 0.2593 \qquad c_0 = 0.8347 \qquad c_1 = -0.3079$

$$\begin{bmatrix} z(-3) \\ z(-2) \\ z(-1) \\ z(0) \\ z(1) \\ z(2) \\ z(3) \end{bmatrix} = \begin{bmatrix} x(-2) & x(-3) & 0 \\ x(-1) & x(-2) & x(-3) \\ x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \\ x(3) & x(2) & x(1) \\ 0 & x(3) & x(2) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

$z(k) = 0.1613, 0.1678, 0.0, 1.0, 0.0, -0.1807, 0.1143$

largest sample magnitude contributing to ISI $= 0.1807$

sum of ISI magnitudes $= 0.6241$

$$\therefore \quad SNR_T = \frac{A^2\left(1-e^{-T/RC}\right)^2}{N_o/4RC}$$

max. SNR_T w.r.t. RC

$$SNR_T = \frac{4A^2 \cdot T}{N_o} \cdot \frac{\left(1-e^{-T/RC}\right)^2}{T/RC}$$

$$\downarrow$$

$$\frac{\left(1-e^{x}\right)^2}{x^2}$$

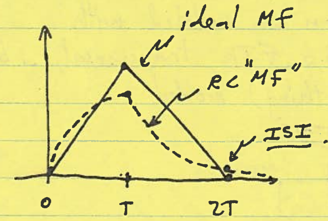
$$\frac{d}{dx}\left[\frac{\left(1-e^{x}\right)^2}{x^2}\right] = 0 = \frac{2x\, e^{-x}\left(1-e^{-x}\right)-\left(1-e^{-x}\right)^2}{x^2} \ \ldots$$

...gives $\quad 1+2x = e^{x} \quad \Rightarrow x \approx 1.257 = \frac{T}{RC}$

$$\therefore \ SNR_T \approx 0.815 \cdot \frac{A^2 T}{N_o/2}$$

$\approx$ 80% of optimal M.F. design ... not bad

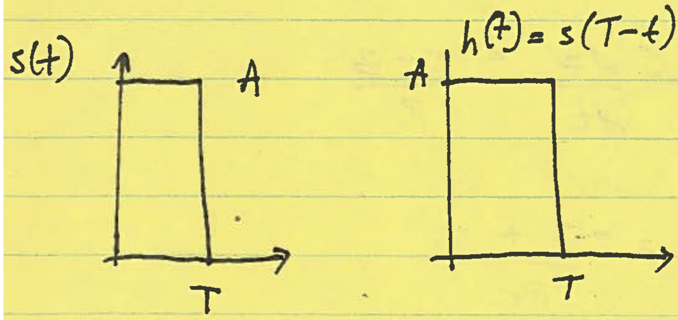
- what is ignored however is the ISI introduced by such a filter



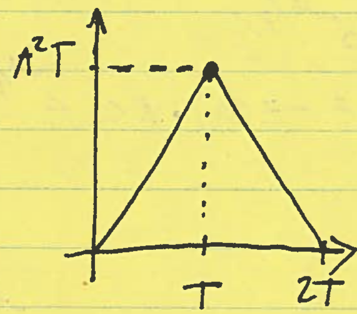
... would you be able to characterize how bad this ISI is?

- in this (RC "MF") case we didn't quite have a perfect MF to operate on a Nyquist pulse & got ISI
- conversley if you have a perfect M.F., but your working pulse is not Nyquist (i.e. the filter is matched to a non-Nyquist pulse) you will get ISI as well

9. MF output with rectangular pulse inputs.



$$z(t) = s(t) * h(t) = \int_0^t s(\tau)\, h(t-\tau)\, d\tau$$

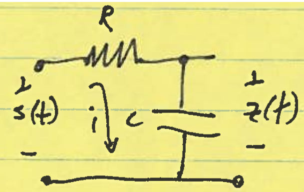


$$z(t) = \begin{cases} A^2 t & 0 \le t \le T \\ -A^2 t + 2A^2 T & T \le t \le 2T \\ 0 & \text{otherwise} \end{cases}$$

$$SNR_T = \frac{A^2 T}{N_0/2} = \frac{\int_{-\infty}^{\infty} |S(f)|^2\, df}{N_0/2}$$

$$= \frac{2A^2 T}{N_0}$$

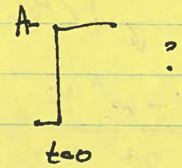
10. MF replaced with RC.



$$i = C\frac{dz}{dt} = \frac{s-z}{R}$$

$$\frac{dz}{dt} = \frac{s-z}{RC}$$

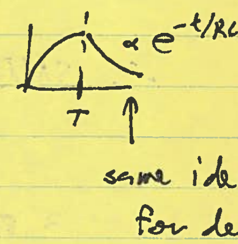
step response



do Laplace $\quad sZ = \frac{A}{sRC} - \frac{Z}{RC}$

$$Z(s) = \frac{A}{s(1+sRC)} = \frac{A}{s} - \frac{A\cdot RC}{(1+sRC)}$$

$$z(t) = \begin{cases} A(1-e^{-t/RC}) & 0 \le t \le T \\ A(1-e^{-T/RC})\,e^{-\frac{(t-T)}{RC}} & T \le t \end{cases}$$

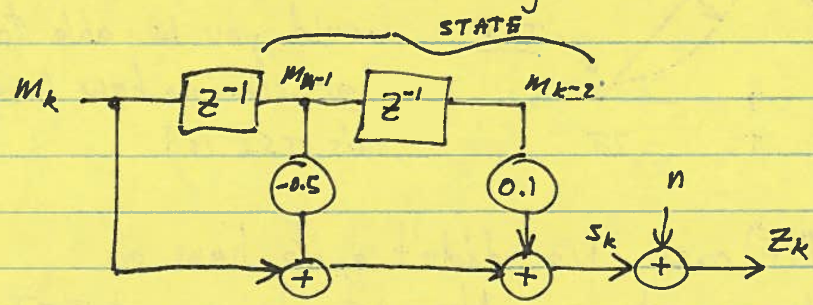


max. at T: $z(T) = A(1-e^{-T/RC})$

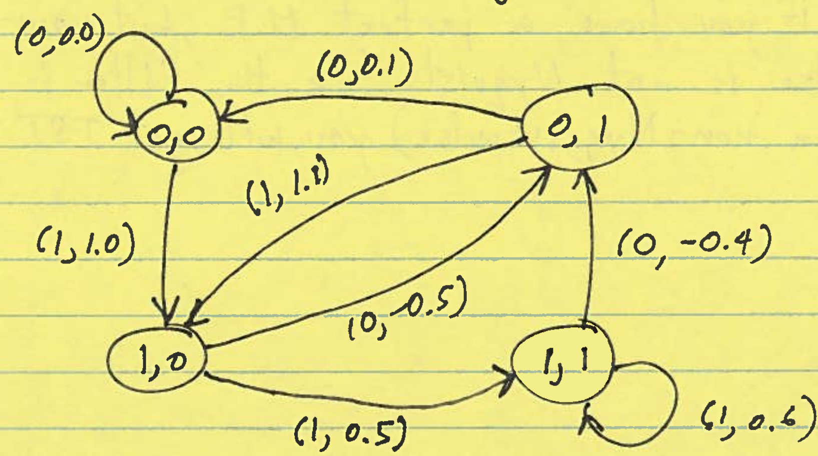
avg. noise o/p power is $\displaystyle\int_{-\infty}^{\infty} \frac{N_0}{2}\cdot\frac{df}{1+(2\pi f RC)^2}\,df = \frac{N_0}{4RC}$

11. Sequence detector ideas.

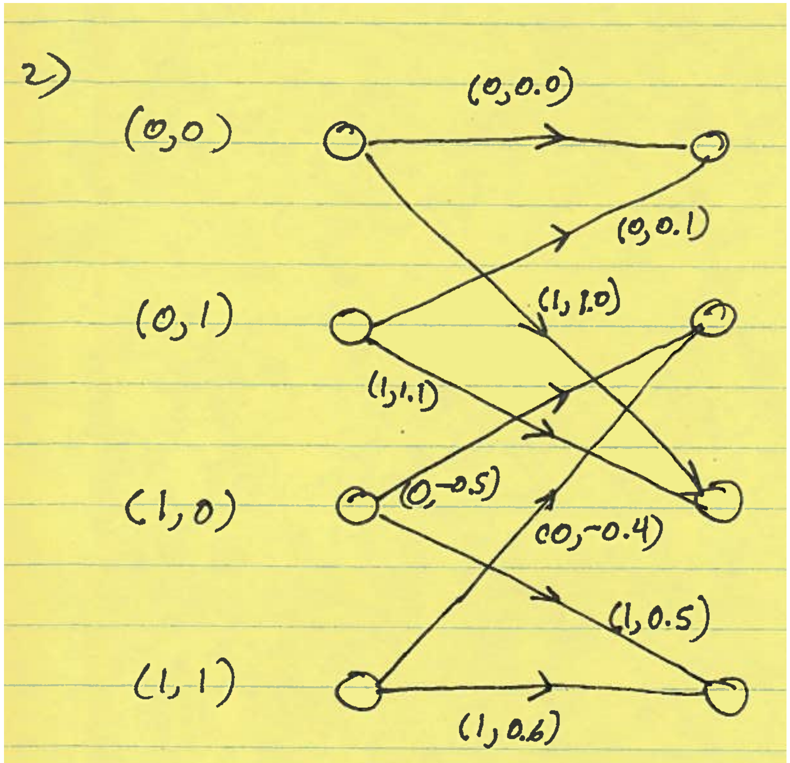
1.) our net channel can be modeled with the shift-register (i.e. FIR, transversal, ... bunch of names for the same thing) model.

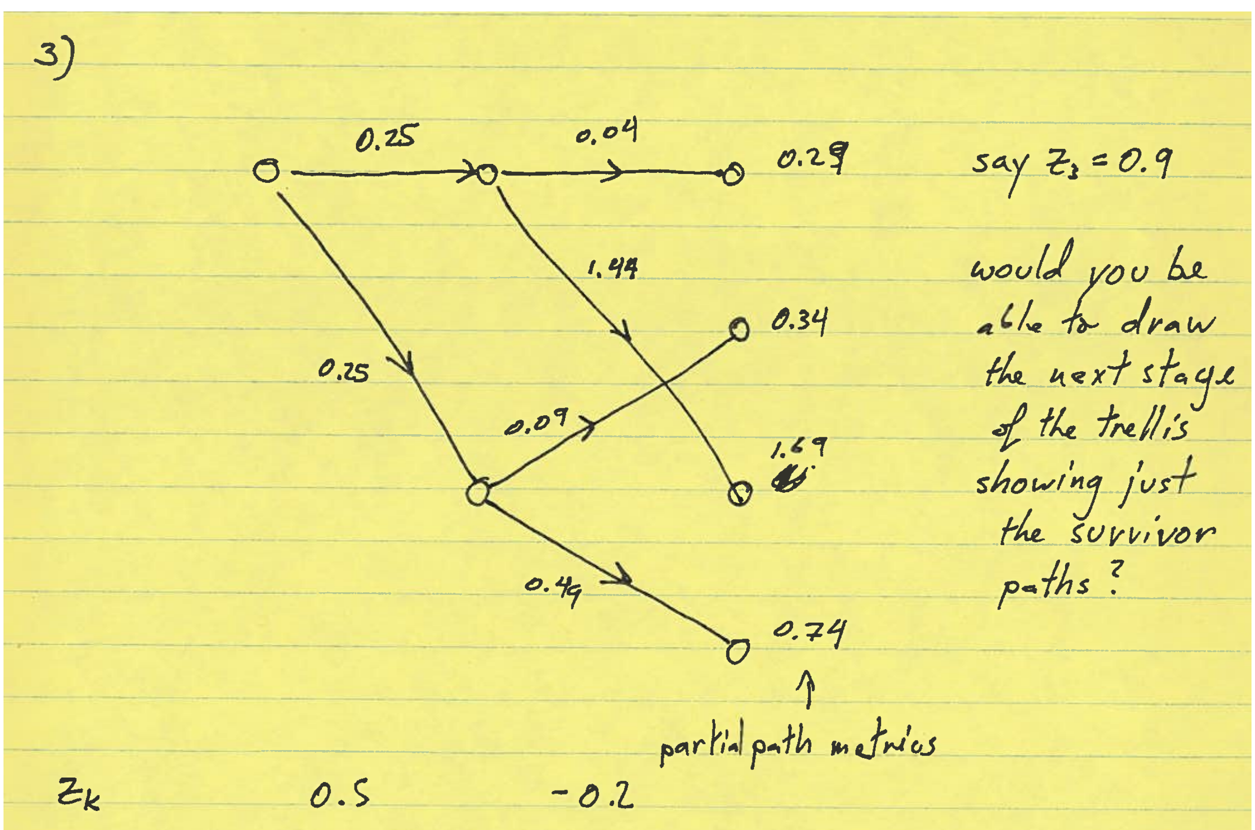


the state transition diagram is



labels: (m_{k-1}, m_{k-2}) } (m_k, s_k)


2)
(0,0)
(0,1)
(1,0)
(1,1)
(0,0.0)
(0,0.1)
(1,1.0)
(1,1.1)
(0,-0.5)
(0,-0.4)
(1,0.5)
(1,0.6)


3)
0.25
0.04
0.29
1.44
0.25
0.34
0.09
1.69
0.49
0.74
partial path metrics
zk
0.5
-0.2
say z3 = 0.9
would you be able to draw the next stage of the trellis showing just the survivor paths?