1. Problem 3.1

a) $f_1 = f_2$, $\phi_1 = \phi_2$ $\int_{-1.5T_2}^{1.5T_2} s_1^2(t)\,dt \neq 0$ ($\cancel{\cos}$ $\int \cos^2 x\,dx \neq 0$)

$\therefore$ not orthogonal

b) $f_1 = \frac{f_2}{3}$, $\phi_1 = \phi_2 = 0$ (say)

$$\int_{-1.5T_2}^{1.5T_2} s_1(t)\,s_2(t)\,dt = \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \cos 2\pi\left(\tfrac{2}{3}f_2\right)t\,dt + \frac{1}{2}\int_{-1.5T_2}^{1.5T_2} \cos 2\pi\left(\tfrac{4f_2}{3}\right)t\,dt$$

$$= \frac{1}{2}\left[\frac{\sin \frac{4}{3}\pi t/T_2}{\frac{4}{3}\pi/T_2}\right]_{-1.5T_2}^{1.5T_2} + \frac{1}{2}\left[\frac{\sin \frac{8}{3}\pi t/T_2}{\frac{8}{3}\pi/T_2}\right]_{-1.5T_2}^{1.5T_2}$$

$$= \frac{\sin 2\pi}{\frac{4}{3}\pi\left(\frac{1}{T_2}\right)} + \frac{\sin 4\pi}{\frac{8}{3}\pi\left(\frac{1}{T_2}\right)} = 0$$

$\therefore$ orthogonal

c) $f_1 = 2f_2$ $\phi_1 = \phi_2 = 0$ (say)

ans: orthogonal

d) $f_1 = \pi f_2$ $\phi_1 = \phi_2 = 0$ (say)

$$\int s_1 \cdot s_2\,dt = \frac{1}{2}\int_a^b \cos(\pi - 1)\,2\pi f_2 t\,dt + \frac{1}{2}\int_a^b \cos(\pi + 1)\,2\pi f_2 t\,dt$$

$\neq 0$ $\therefore$ not orthogonal

e) $f_1 = f_2$, $\phi_1 = \phi_2 + \pi/2$

$$\int_a^b \sin(2\pi f_2 t)\cos(2\pi f_2 t)\,dt = 0 \quad \therefore \text{orthogonal}$$

f) $f_1 = f_2$, $\phi_1 = \phi_2 + \pi$ ans: not orthogonal

2. Problem 3.2

a) $$\int_{-2}^{2} \psi_1(t)\,\psi_2(t)\,dt = \int_{-2}^{-1}(-A)(-A)\,dt + \int_{-1}^{0}(A)(-A)\,dt + \int_{0}^{1}(A)(A)\,dt + \int_{1}^{2}(-A)(A)\,dt$$

$$= A^2 - A^2 + A^2 - A^2 = 0$$

$$\int_{-2}^{2} \psi_1(t)\,\psi_3(t)\,dt = \int_{-2}^{-1}(-A)(-A)\,dt + \int_{-1}^{0}(A)(-A)\,dt + \int_{0}^{1}(A)(-A)\,dt + \int_{1}^{2}(-A)(-A)\,dt$$

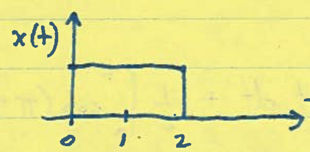
$$= A^2 - A^2 - A^2 + A^2 = 0$$

$$\int_{-2}^{2} \psi_2(t)\,\psi_3(t)\,dt = \int_{-2}^{0}(-A)(-A)\,dt + \int_{0}^{2}(A)(-A)\,dt$$

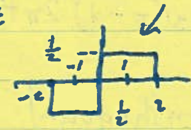
$$= 2A^2 - 2A^2 = 0$$

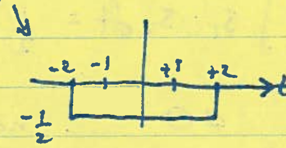
b) $$\int_{-2}^{2} \psi_3^2(t)\,dt = \int_{-2}^{2} A^2\,dt = \left[A^2 t\right]_{-2}^{2} = 2A^2 + 2A^2 = 4A^2$$

$\therefore$ to be orthonormal $4A^2 = 1$ $\therefore A = \frac{1}{2}$

c) 

$x(t) = \psi_2(t) - \psi_3(t)$ where





3. Problem 3.10

a) $$R_s = \frac{9600 \text{ bps}}{3 \text{ bits/symbol}} = 3200 \text{ symbols / second}$$

b) $$r = \frac{W - W_0}{W_0}$$ W_0 = Nyquist min. bw

$$W_0 = \frac{R_s}{2} = 1600 \text{ Hz}$$

$$r = \frac{2400 - 1600}{1600} = 0.5$$

4. Problem 4.1

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$ in BPSK $s_1 = A\cos\omega_c t$, $s_2 = -A\cos\omega_c t$

$$E_d = \int_{-\infty}^{\infty}(s_1 - s_2)^2 dt = \int_0^T (2A\cos\omega_c t)^2 dt$$
$$= 2A^2T$$

$$E_b = \frac{1}{2}\int_0^T (A\cos\omega_c t)^2 dt + \frac{1}{2}\int_0^T (-A\cos\omega_c t)^2 dt$$
$$= \frac{A^2T}{2}$$

$$\therefore P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

$A = 1\,mV$, $T = \frac{1}{5000}\,s$, $N_0 = 10^{-11}\,\frac{W}{Hz}$

$$P_B = Q\left(\sqrt{\frac{10^{-6}}{5000\times 10^{-11}}}\right) = Q(\sqrt{20}) = Q(4.47)$$

for $x > 3$ $\quad Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$$\therefore P_B \approx \frac{1}{\sqrt{40\pi}} e^{-10} = 4.05\times 10^{-6}$$

avg. no. of bit errors in one day =

$$5000\,\frac{\text{bit}}{\text{sec.}} \times 86400\,\frac{\text{sec.}}{\text{day}} \times 4.05\times 10^{-6} = 1750$$

5. Problem 4.2

a) $1000 \dfrac{\text{bit}}{\text{sec.}} \times 86400 \dfrac{\text{sec}}{\text{day}} = 8.64\times10^{7} \dfrac{\text{bit}}{\text{day}}$

$P_B = \dfrac{100}{8.64\times10^{7}} = 1.16\times10^{-6} \leftarrow$ effective ~~required~~ BER

b) $P_B = Q\left(\sqrt{\dfrac{2E_b}{N_0}}\right) = Q\left(\sqrt{\dfrac{2\overbrace{P}^{10^{-6}W}\cdot \overbrace{T}^{\frac{1}{1000}s}}{N_0}}\right) = Q\left(\sqrt{\dfrac{2\times10^{-6}\times\frac{1}{1000}}{10^{-10}}}\right) = Q(\sqrt{20})$

$= Q(4.47) = 4.05\times10^{-6}$ ... this power is NOT enough

6. Problem 4.3

noncoherent BFSK : $\dfrac{E_b}{N_0} = 13\text{dB} = 19.95$

$P_B = \dfrac{1}{2} e^{-E_b/2N_0} = 2.32\times10^{-5}$

coherent BPSK : $\dfrac{E_b}{N_0} = 8\text{dB} = 6.31$

$P_B = Q\left(\sqrt{\dfrac{2E_b}{N_0}}\right) \doteq Q\left(\sqrt{2\times6.31}\right) = Q(3.55)$

$\simeq \dfrac{1}{\sqrt{2\pi\times12.6}} e^{-\frac{12.6}{2}} = 2.07\times10^{-4}$

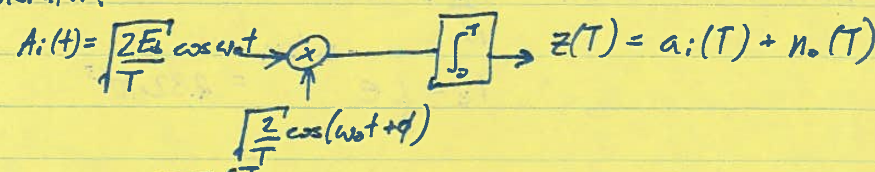
$\therefore$ select noncoherent BFSK

7. Problem 4.4

$m(k)$ =	1	0	1	0	1	0	1	1	1
(4.43) with 1 as start	0	0	1	1	0	0	1	0	1
(4.43) with 0 as start	1	1	0	0	1	1	0	1	0
(4.44) with 1 as start	1	0	0	1	1	0	0	0	0
(4.44) with 0 as start	0	1	1	0	0	1	1	1	1

8. Problem 4.6

a) in general...



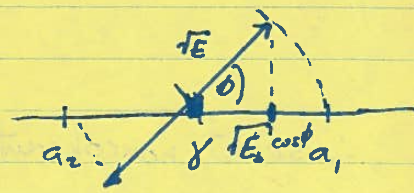
$$a_1(T) = \frac{2}{T}\sqrt{E_b}\int_0^T \cos w_0 t \cdot \cos(w_0 t + \phi)\,dt$$

$$= \frac{2}{T}\sqrt{E_b}\int_0^T \frac{1}{2}\left[\cos\phi + \cos(2w_0 t + \phi)\right] dt$$

$$\doteq \sqrt{E_b}\cos\phi$$

similarly $a_2(T) = -\sqrt{E_b}\cos\phi$

$\therefore\ a_1 - a_2 = 2\sqrt{E_b}\cos\phi$



$\therefore$ recall $\sigma_0^2 = N_0/2$ (noise o/p at sampling instant from M.F. or autocorrelator)

$$\therefore\ P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}} \cdot \cos\phi\right)$$

for $\frac{E_b}{N_0} = 9.6\text{ dB} = 9.12$ & $\cos 25° = 0.9063$

$$P_B = Q(\sqrt{18.24} \cdot 0.9063) = Q(3.87)$$

$$P_B \approx \frac{1}{3.87\sqrt{2\pi}}\, e^{-\frac{(3.87)^2}{2}} = 5.8\times10^{-5}$$ (without $\phi < 0°$ $= 9.7\times10^{-6}$)

b) $P_B = 10^{-3} = \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}} \Rightarrow x = 3.12 = \sqrt{2E_b/N_0} \cdot \cos\phi$

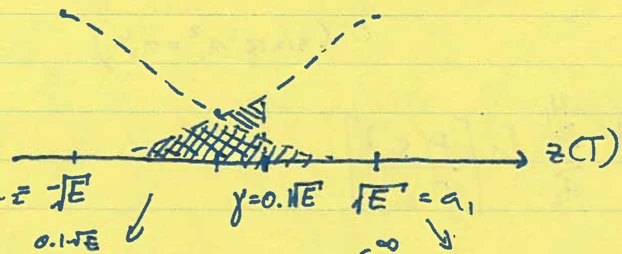
$\cos\phi = 3.12/\sqrt{18.24} = 0.73 \quad \therefore\ \phi = 43°$

9. Problem 4.9

a) $P_B = Q\left(\sqrt{\dfrac{2E_b}{N_0}}\right)$; $\dfrac{E_b}{N_0} = 6.8\,dB = 4.786$

$P_B = Q(3.09) \simeq 10^{-3}$

b)



$$P_B = \frac{1}{2}\int_{-\infty}^{0.1\sqrt{E}} p(z|s_1)\,dz + \frac{1}{2}\int_{0.1\sqrt{E}}^{\infty} p(z|s_2)\,dz$$

$$= \frac{1}{2} Q\left(\frac{1.1\sqrt{E}}{\sigma_0}\right) + \frac{1}{2} Q\left(\frac{0.9\sqrt{E}}{\sigma_0}\right)$$

for M.F. $E = E_b$, $\sigma_0^2 = N_0/2$

$$\therefore P_B = \frac{1}{2} Q\left(1.1\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2} Q\left(0.9\sqrt{\frac{2E_b}{N_0}}\right)$$

$E_b/N_0 = 6.8$ dB $= 4.786$

$$P_B = \tfrac{1}{2} Q(1.1 \times 3.09) + \tfrac{1}{2} Q(0.9 \times 3.09)$$

$$= 1.4 \times 10^{-3}$$

c)

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$$\frac{z(a_1 - a_2)}{\sigma_0^2} - \underbrace{\cancel{\frac{(a_1^2 - a_2^2)}{2\sigma_0^2}}}_{0 \text{ (since } a_1^2 = a_2^2)} \underset{H_2}{\overset{H_1}{\gtrless}} \ln\left[\frac{P(s_2)}{P(s_1)}\right]$$

$$\frac{z(2\sqrt{E_b})}{N_0/2} \underset{H_2}{\overset{H_1}{\gtrless}} \ln\left[\frac{P(s_2)}{P(s_1)}\right]$$

$$z \underset{H_2}{\overset{H_1}{\gtrless}} \frac{N_0/2}{2\sqrt{E_b}} \ln\left[\frac{P(s_2)}{P(s_1)}\right] = \gamma$$

$$\gamma = 0.1\sqrt{E_b} = \frac{N_0}{4\sqrt{E_b}} \ln\left[\frac{P(s_2)}{P(s_1)}\right]$$

knowing $\dfrac{E_b}{N_0} = 4.786$ we find $\ln\left[\dfrac{P(s_2)}{P(s_1)}\right] = 1.914$

$$\frac{P(s_2)}{P(s_1)} = \frac{P(s_2)}{1 - P(s_2)} = e^{1.914} = 6.782$$

$\therefore P(s_2) = 0.87$ & $P(s_1) = 0.13$

10. Problem 4.14

a) $r = 1$ $\quad W_{DSB} = (1+r)R_s$

$50 \text{ kHz} = 2R_s$

$R_s = 25 \text{ ksymbols/s}$

$$\frac{R}{R_s} = \frac{100 \text{ kbps}}{25 \text{ ksymbols/s}} = 4 = \log_2 M \quad \therefore M = 16$$

Since a Gray code is used $P_B \approx \dfrac{P_E}{\log_2 M}$

$$\therefore P_E = (\log_2 M) P_B = 4 \times 10^{-3}$$

$$P_E = 2Q\left[\left(\sqrt{\frac{2E_s}{N_0}}\right)\sin\left(\frac{\pi}{M}\right)\right] = 4.0 \times 10^{-3}$$

$$Q(x) = 2 \times 10^{-3} \Rightarrow x = 2.88$$

$$\sqrt{\frac{2 \cdot E_s}{N_0}} \sin\left(\frac{\pi}{M}\right) = 2.88$$

$$\sqrt{\frac{2E_s}{N_0}} = \frac{2.88}{\sin\left(\frac{\pi}{16}\right)} = \frac{2.88}{0.19509} = 14.76$$

$$E_s / N_0 = 108.9 = 20.4 \text{ dB}$$

b) $$\frac{E_b}{N_0} = \frac{108.9}{k} = \frac{108.9}{4} = 27.2 = 14.3 \text{ dB}$$