L12: Equalization

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Outline

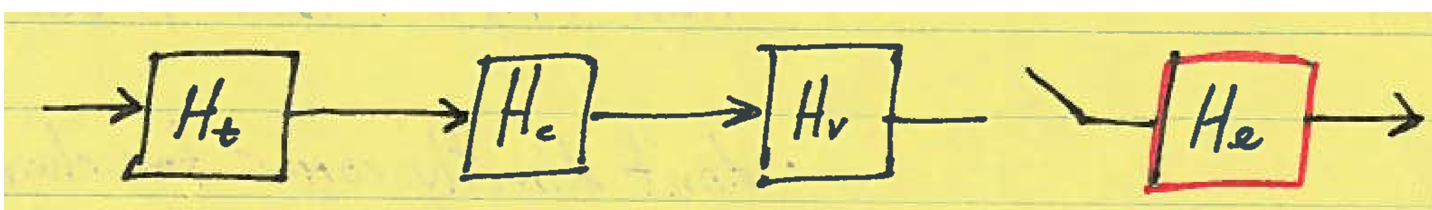
- Intro to Equalization
- ZFE
  - Zero-Forcing Equalizer
- MMSE
  - Minimum-Mean-Squared Equalizer
- DFE
  - Decision-Feedback Equalizer

1.2 Overview

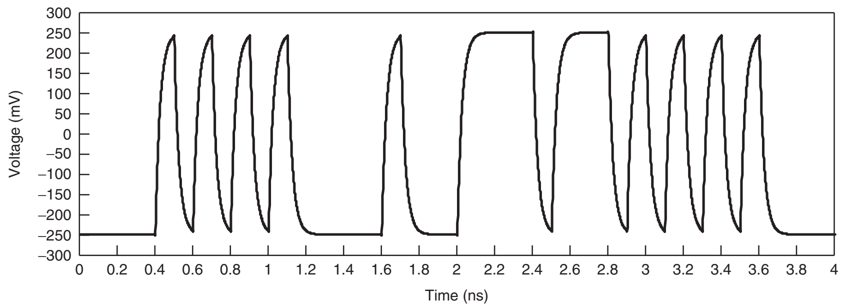
- To maximize SNR and eliminate ISI set

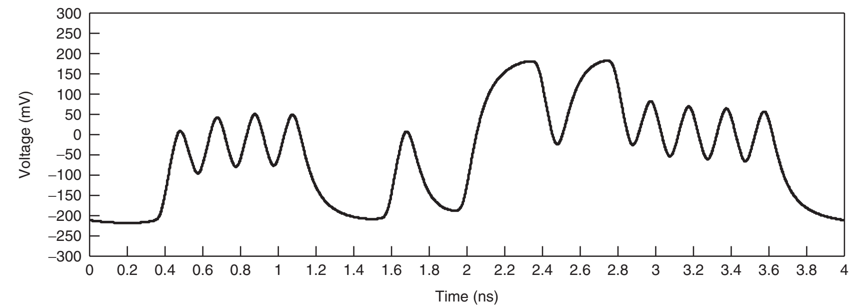
$$|H_r(f)| = \frac{\alpha |P_r|^{\frac{1}{2}}}{G_n^{1/4} |H_c|^{\frac{1}{2}}} \qquad |H_t| = \frac{(A/\alpha) |P_r|^{\frac{1}{2}} G_n^{1/4}}{|H_c|^{1/2}}$$

- These may be hard to implement
  - Let TX and RX focus on pulse shaping & noise
- Introduce another filter to focus on ISI
  - The equalizer

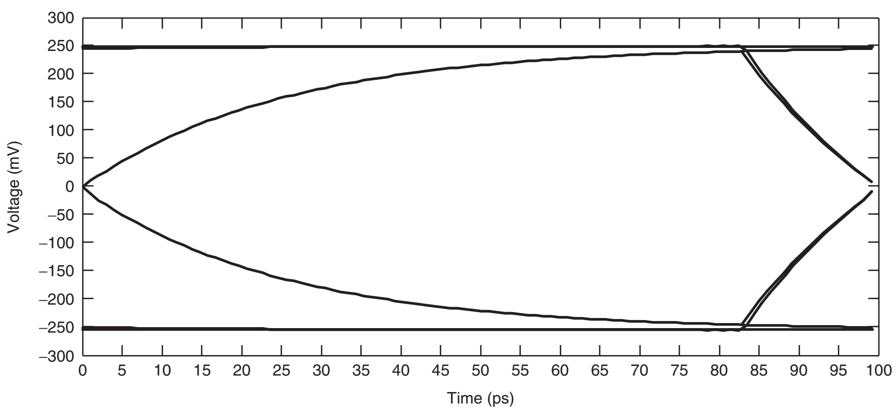


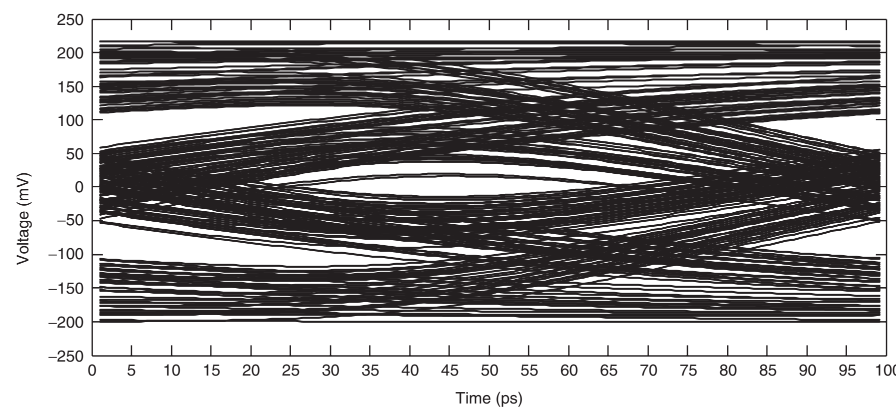
Overview





- Eye diagrams



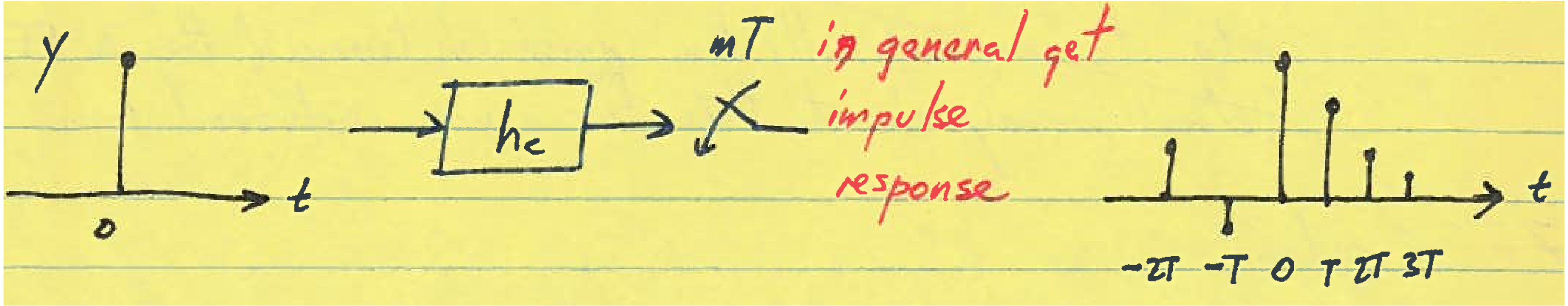


12.2 Equalizer Types

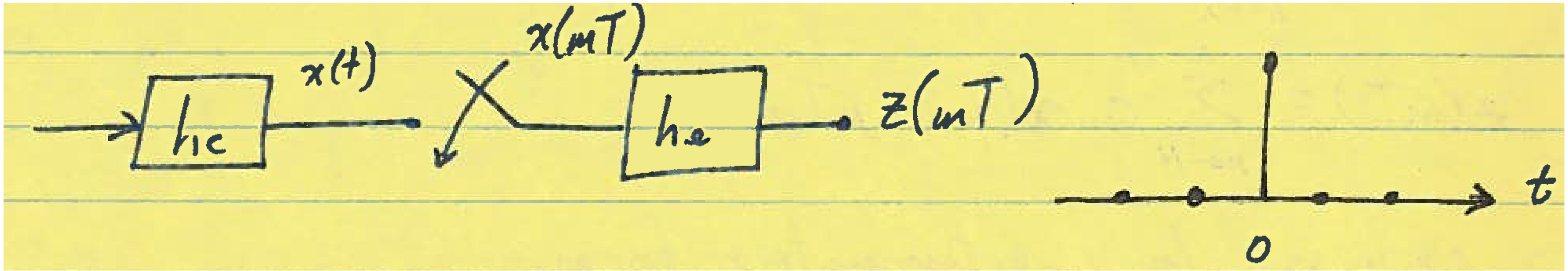
- Filter-Based
  - Linear (Transversal Filters)
    - ZFE
    - MMSE
  - Non-Linear
    - DFE
- Other
  - Maximum-Likelihood Sequence Estimation (MLSE)
    - Viterbi

12.3 Zero-Forcing Equalizer (ZFE)

- If this is the impulse response before your detector…

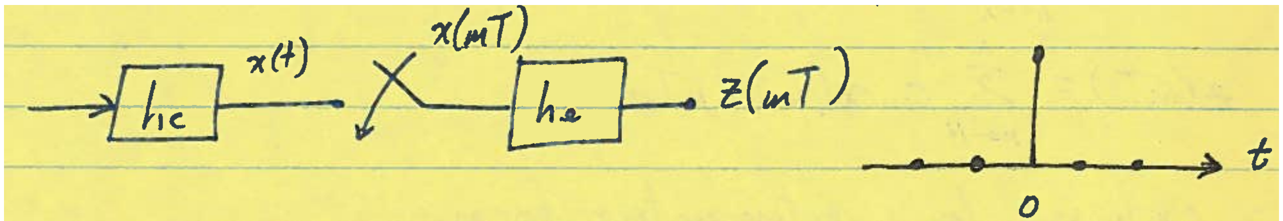


- The ZFE attempts to achieve…



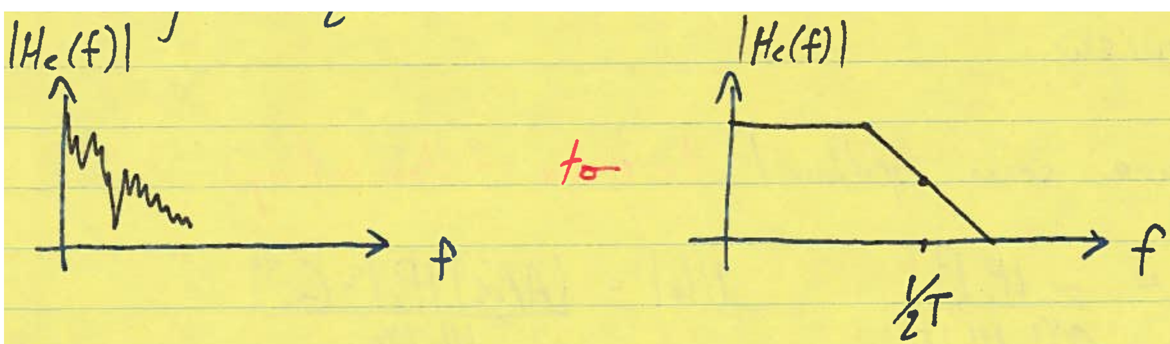
ZFE Response

- This response is effectively achieved by…



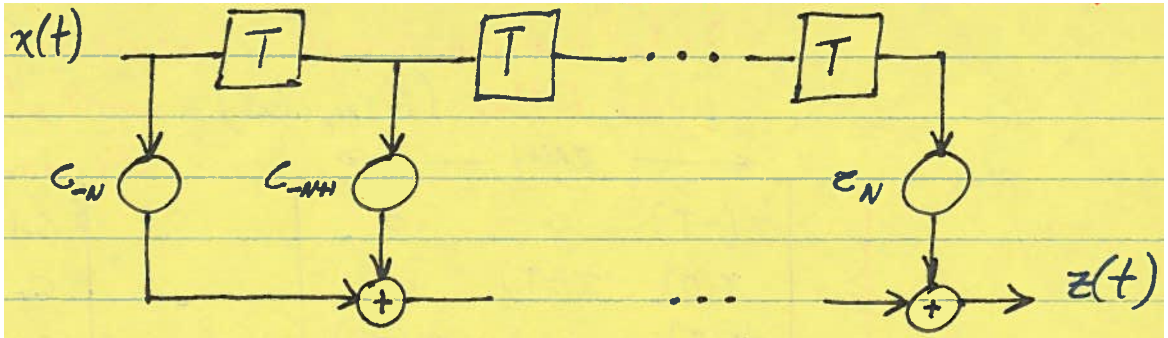
- …inverting the channel

$$H_e(f) = \frac{1}{H_c(f)} = \frac{1}{|H_c(f)|} \cdot e^{-j\theta_c(f)}$$

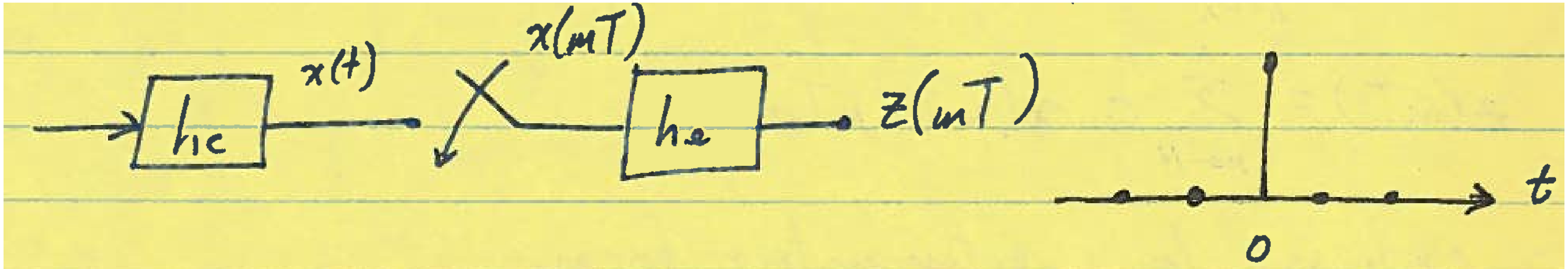


ZFE Filter Structure

- This channel inversion can be achieved with…
  - FIR filter structure (transversal filter)

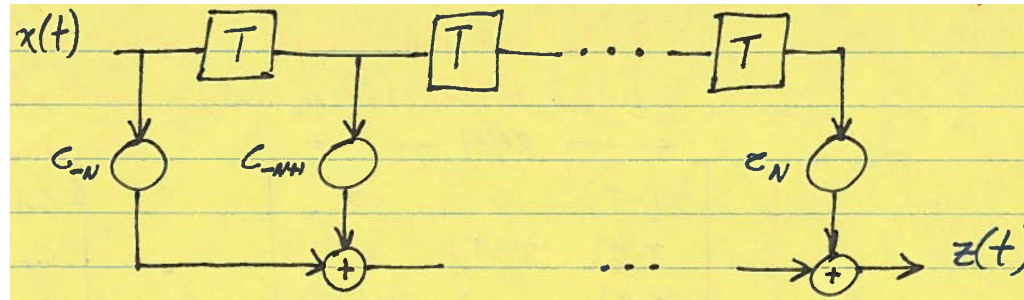


  - x is the equalizer input (with ISI)
  - z is the equalizer output (without ISI)



ZFE: Formal Signal Description

- Math for FIR filter input/output relation…



$$z(t) = \sum_{n=-N}^{N} c_n x(t - nT)$$

$$z(mT) = \sum_{n=-N}^{N} c_n x(mT - nT)$$

- Convenient to describe WHOLE ZFE impulse response
  - i.e. the whole of z for any particular channel input

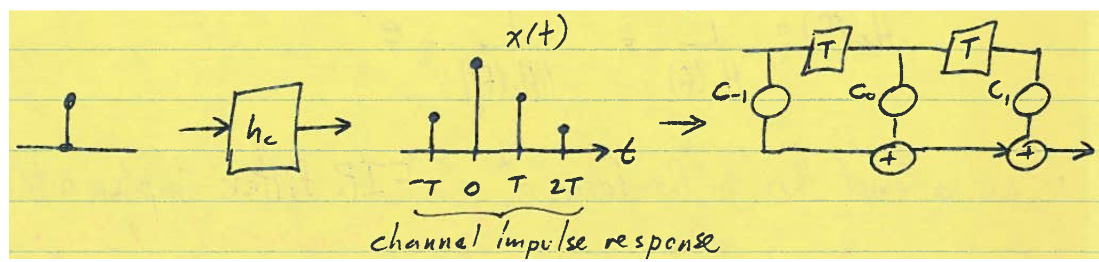
ZFE Channel Response

- Capture entire ZFE response
  - using convolution matrix **X**

$$\bar{z} = \bar{X} \cdot \bar{c}$$

- **X**
  - state of equalizer as a function of time
    - state: the signal value at each tap

- For…
  - a channel impulse response of K impulses
  - into an equalizer of N+1 taps



- We have a **z** how long?

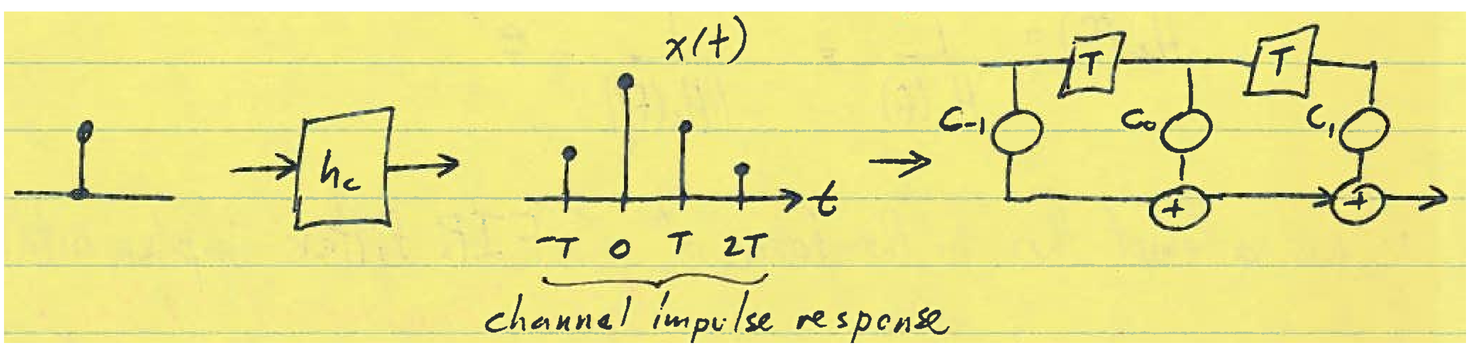
ZFE Channel Response

- $\bar{z} = \bar{X} \cdot \bar{c}$

time = $K+2N$

$$
\begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}
=
\begin{bmatrix}
x(-T) & 0 & 0 \\
x(0) & x(-T) & 0 \\
x(T) & x(0) & x(-T) \\
x(2T) & x(T) & x(0) \\
0 & x(2T) & x(T) \\
0 & 0 & x(2T)
\end{bmatrix}
\cdot
\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}
$$

$\longleftarrow \; 2N+1 \; \longrightarrow$



Desired Response

- What output do you want from equalizer?

$$\bar{z} = \bar{X} \cdot \bar{c}$$

↑ your desired output vector: a 1 and the rest zeros

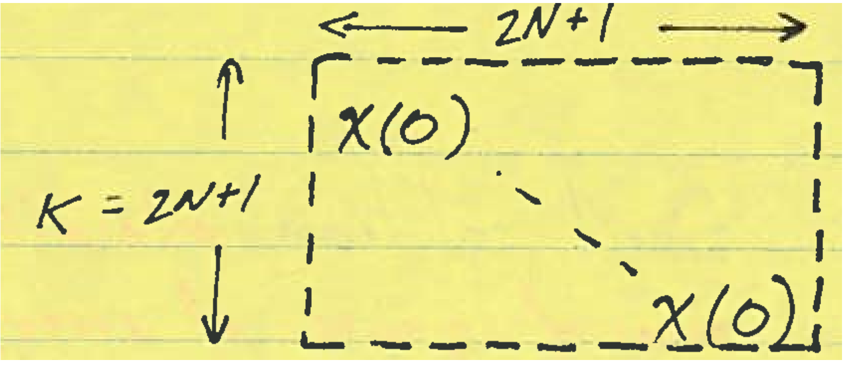
- And how do you get this?
  - using math

$$\bar{c} = \bar{X}^{-1} \bar{z}$$

- And what if **X** is not a square matrix?

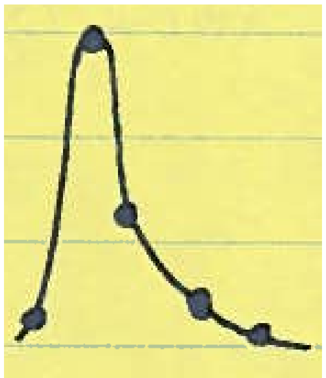
ZFE Coefficients

- Book approach…
- …Just make ZFE square!



ZFE Example

- Channel response:



$$\begin{matrix} x(-1) \\ x(0) \\ x(1) \\ x(2) \\ x(3) \end{matrix} \begin{bmatrix} 36 \\ 230 \\ 97 \\ 37 \\ 18 \end{bmatrix} mV = \bar{x}$$

- Say you want:

$$\bar{z} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer

- Square convolution matrix:

$$\bar{X} = \begin{bmatrix} 230 & 36 & 0 & 0 & 0 \\ 97 & 230 & 36 & 0 & 0 \\ 37 & 97 & 230 & 36 & 0 \\ 18 & 37 & 97 & 230 & 36 \\ 0 & 18 & 37 & 97 & 230 \end{bmatrix}$$

$$\bar{c} = \bar{X}^{-1} \bar{z} = \begin{bmatrix} -0.77 \\ 4.93 \\ -1.98 \\ 0.14 \\ -0.13 \end{bmatrix}$$

normalize using $\frac{c_i}{\sum c_i}$

$$\begin{bmatrix} -.097 \\ .624 \\ -.25 \\ .017 \\ .016 \end{bmatrix}$$

ZFE-LS

- Least-squares (LS)
  - A standard approach to the solution of over determined systems

$$\bar{c}_{LS} = \arg\min_{\bar{c}} \left\| \bar{z} - \bar{X}\bar{c} \right\|^2$$

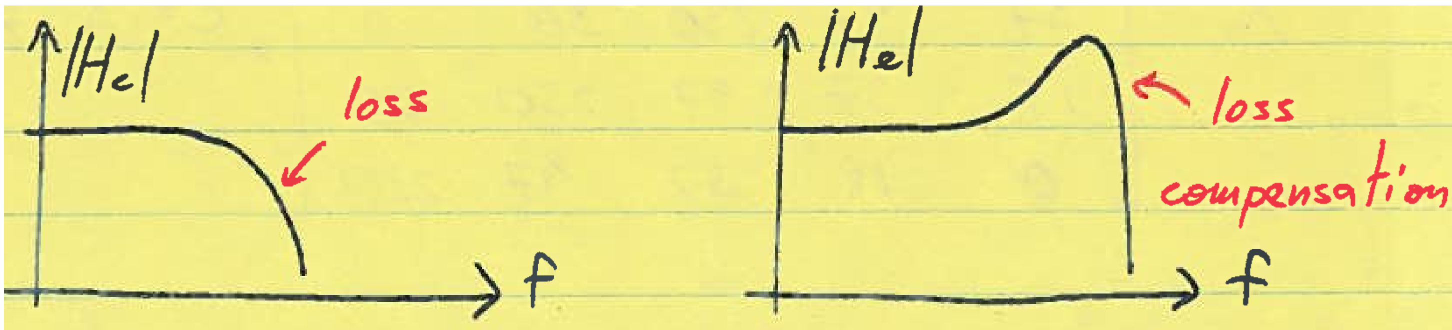
$$\sum_i \left(z_i - \sum_j x_{ij} c_j \right)^2$$

  - minimize difference-squared between z and product of chosen c with x

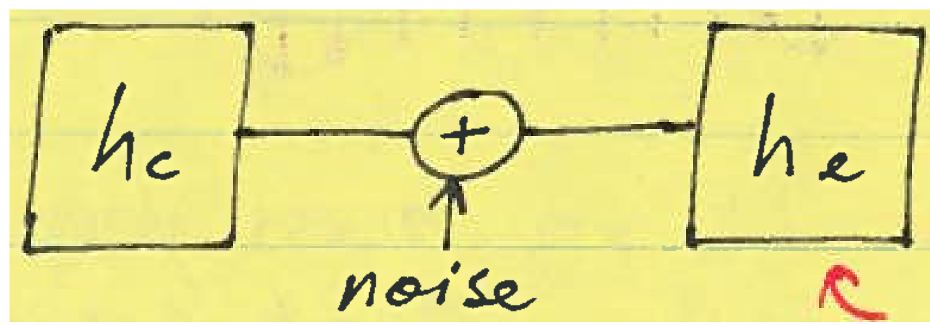
$$\bar{c}_{LS} = \left(\bar{X}^T \cdot \bar{X} \right)^{-1} \bar{X}^T \cdot \bar{z}$$

Noise Enhancement

- ZFE and ZFE-LS is SINGLE-MINDED!
- In compensating only to remove ISI…



- …noise is amplified
  - noise enhancement
- You compensate to get back your original signal
  - …and in the process you increase the net noise into detector



12.5 Minimum-Mean-Squared Equalizer

- Seek a compromise between ISI and noise enhancement
- Instead of least-squares to minimize ISI…

$$\bar{c}_{LS} = \bar{c}_{ZFE} = \left(\bar{X}^T X\right)^{-1} \bar{X}^T \bar{z}$$

- …account for SNR

identity matrix

$$\bar{c}_{MMSE} = \left(\frac{I}{SNR} + \bar{X}^T X\right)^{-1} \bar{X}^T \bar{z}$$

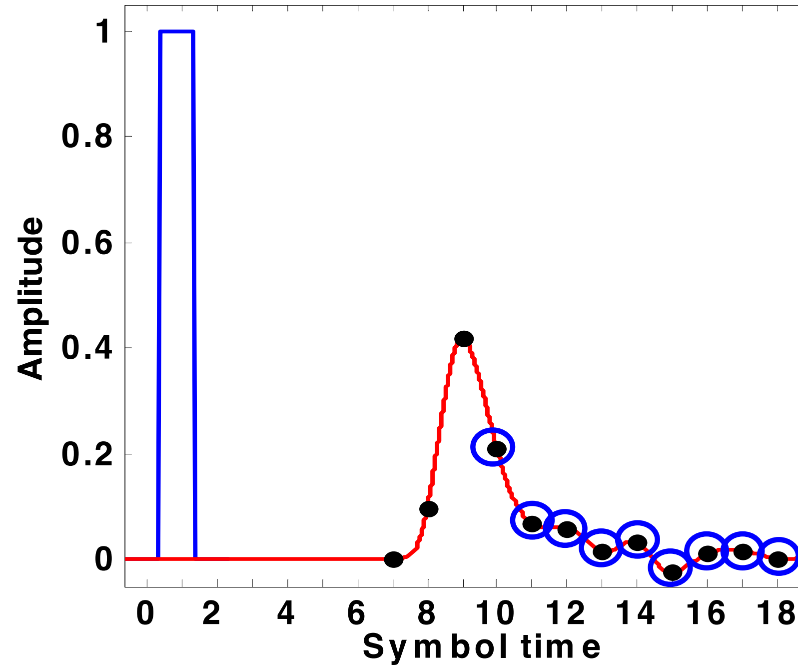
MMSE

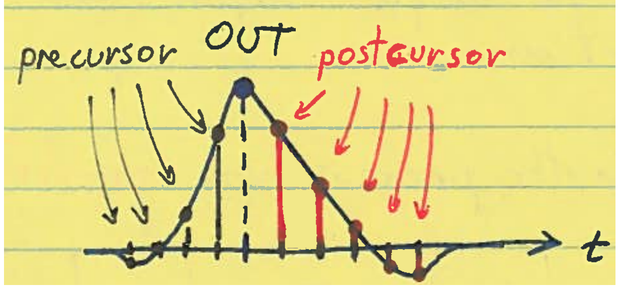
- MMSE allows a bit of ISI
  - but has less noise enhancement

- MMSE not straightforward to apply
  - noise may not be known
  - harder to make adaptive

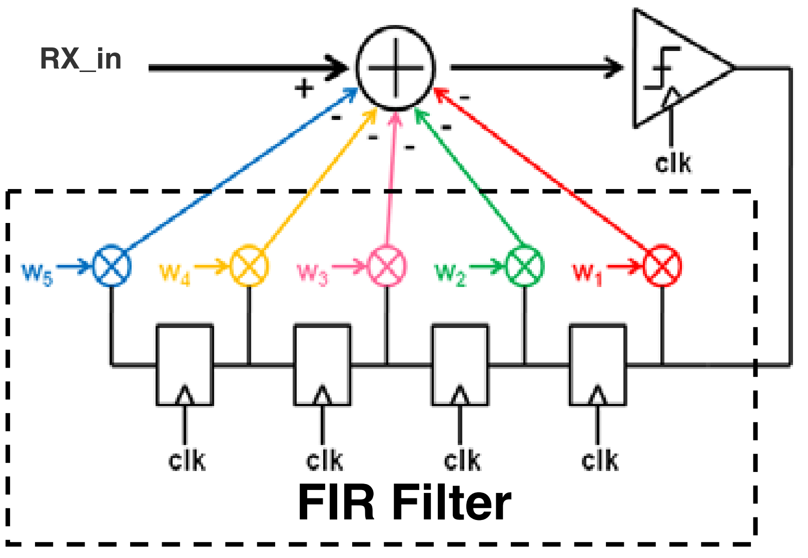
12.6 Decision-Feedback Equalizer (DFE)

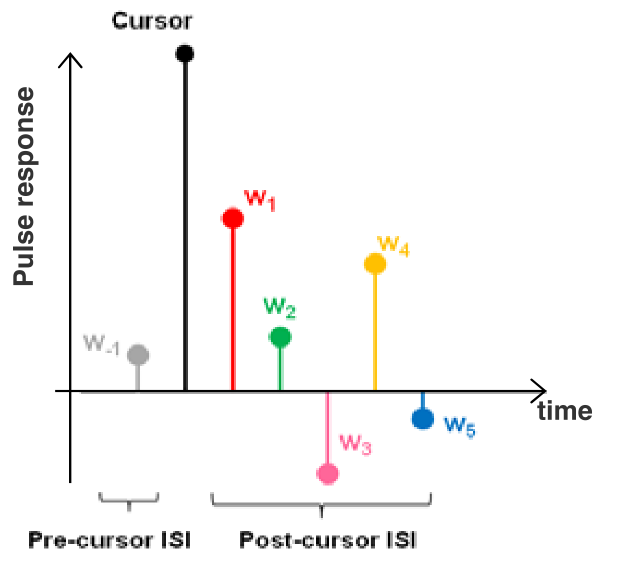
- A non-linear approach
- If you know which bit is transmitted you know exactly what ISI it will cause (in postcursor)
- Why not just directly cancel the coming ISI?





DFE





- Key advantage: no noise enhancement

DFE

- Only handles postcursor
  - May need linear filter for remove precursor