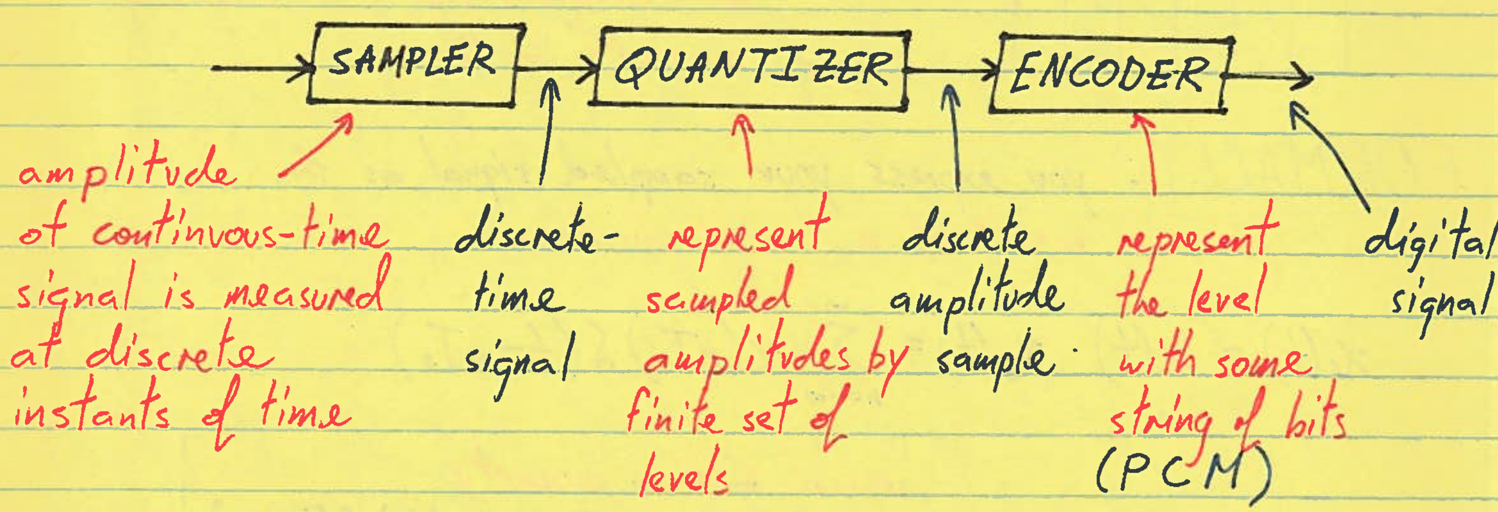
L5 Analog-to-Digital Conversion

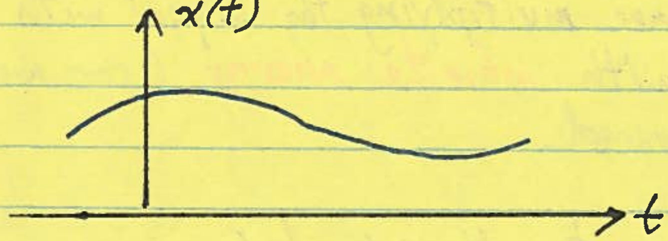
- Sending analog signals in digital format
- A-to-D, A/D, ADC, data converters accomplish this change in format
- Here we outline the general principals

5.1 General Architecture



5.2 Impulse Sampling

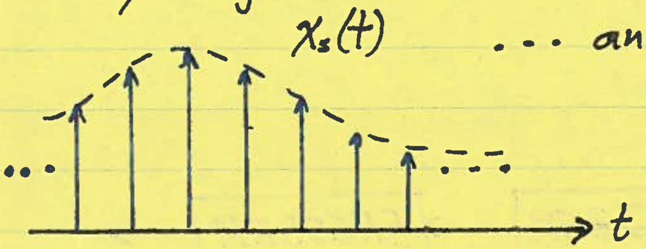
- The easiest way to start ~~quan~~ quantifying the properties of the A/D is to model its sampler in terms of Dirac δ's
- Assume you have arbitrary analog signal



- You sample uniformly every T_s

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

- Then you get . . .



. . . an infinite sequence of samples: $x(nT_s)$ ← your original signal at discrete moments in time

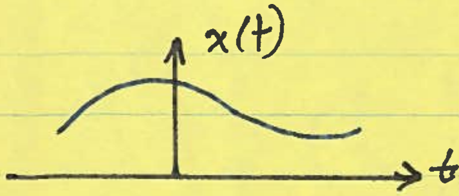
FORMALLY: you express your sampled signal as the result of a multiplication

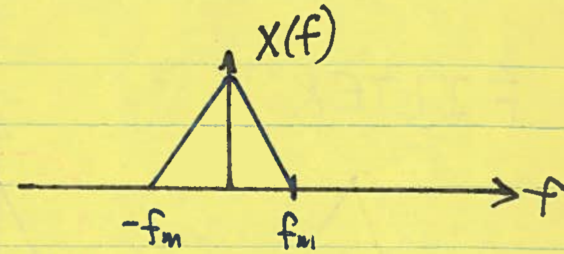
$$x_s(t) = x(t) \cdot x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$$

[recall ~~sifting~~ property
$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$]

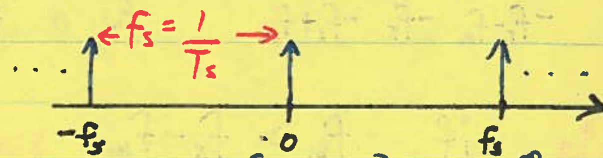
5.3 The Sampling Theorem

- With sampling aren't we just getting partial results?
  - we're missing so much of the wave
- At least mathematically we are multiplying the signal with a periodic pulse train... with infinite energy (so we have that "content" part covered
- But the key is to coherently capture the signal in our samples ... that depends on sample rate

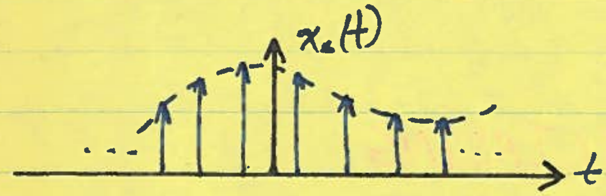


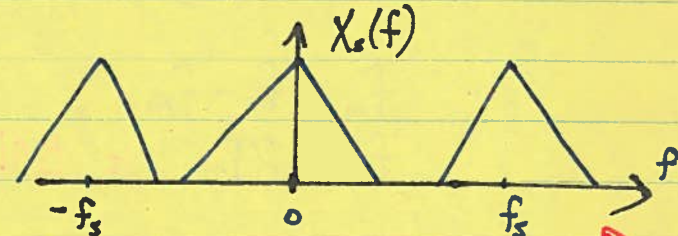


$$x_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



$$F.T.\{x_s(t)\} = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$





$$x_s(t) = x(t) \cdot x_s(t)$$
$$= \sum x(nT_s)\delta(t - nT_s)$$

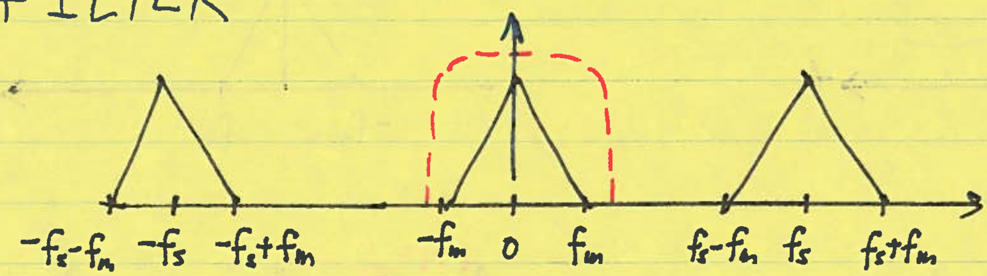
- multiplication in time domain is convolution in freq. domain

$$X_s(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$
$$= f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s)$$
$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$X(f)$ repeats periodically with period f_s

- not only do we get spectrum of original we get a bunch of coples at frequency offset
- to get back our original we need to...

... FILTER



Note: if $f_m > f_s - f_m$ spectra overlap

- this would distort your original signature

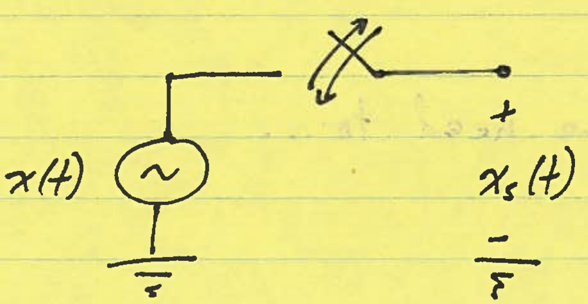
$$f_m > f_s - f_m$$
$$f_s < 2f_m \text{ is BAD} \Rightarrow \text{ALIASING}$$

- To avoid this you need $f_s \geq \underbrace{2f_m}_{\text{Nyquist Rate}}$

- Must sample at or above Nyquist rate
- This is called Nyquist criterion

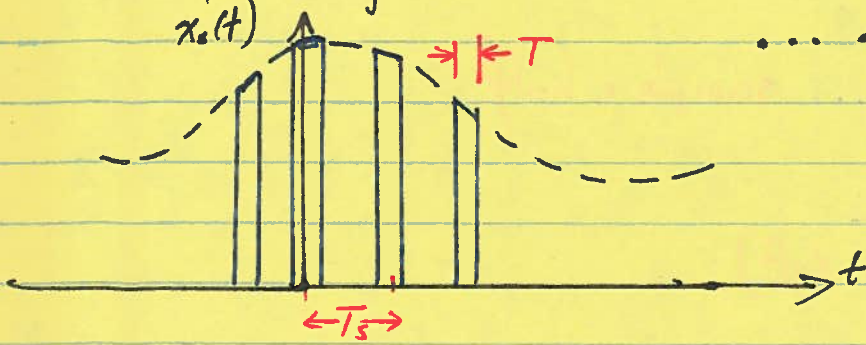
5.4 Natural Sampling

- x_s is very ideal
- More likely to sample for finite amt. of time



where switch is closed for some T seconds

- sampled signal is...

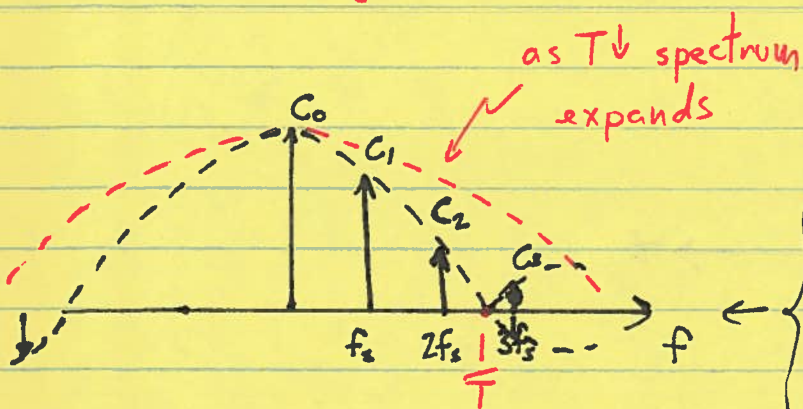


... again treat as multiplication

$$x_s(t) = x(t) \cdot x_p(t)$$

where pulse train takes on the F.S.

$$x_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s t}$$



where $c_n = \dfrac{T}{T_s} \dfrac{\sin(n\pi f_s T)}{n\pi f_s T}$

$= \dfrac{T}{T_s} \text{sinc}(n f_s T)$

$c_n = 0$ when $n\pi f_s T = \pi$

first min. c_n frequency: $n = \dfrac{1}{f_s T}$

$$\left\lfloor \frac{1}{f_s T} \cdot f_s \right\rfloor \le f_{min}|_1 \le \left\lceil \frac{1}{f_s T} \cdot f_s \right\rceil$$

$$\left\lfloor \frac{1}{T} \right\rfloor \le f_{min}|_1 \le \left\lceil \frac{1}{T} \right\rceil$$

the spectral characteristics of our sampled function now?

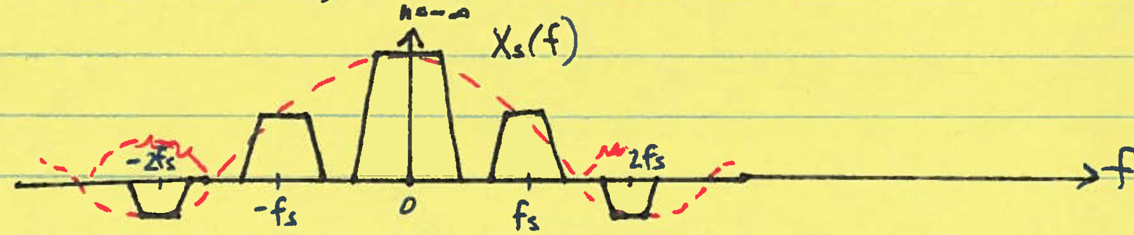
$$X_s(f) = F.T.\left\{ x(t) \sum c_n e^{j2\pi n f_s t} \right\}$$

$$= F.T.\left\{ \sum c_n x(t) e^{j2\pi n f_s t} \right\}$$

recall freq. shifting property of F.T.

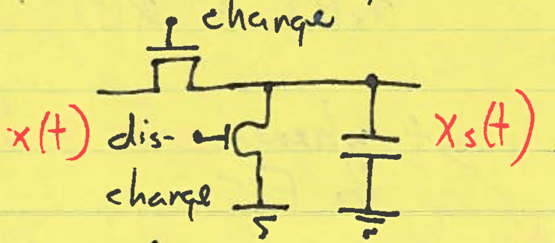
$$x(t) e^{j2\pi f_0 t} \leftrightarrow X(f - f_0)$$

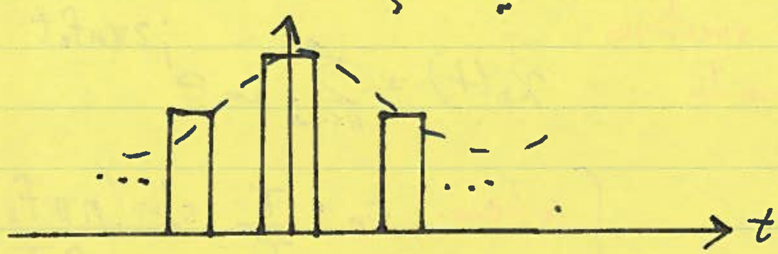
$$\therefore X_s(f) = \sum_{n=-\infty}^{\infty} c_n X(f - n f_s)$$



5.5 Flat-Top Sampling

- even more realistically... sample & hold

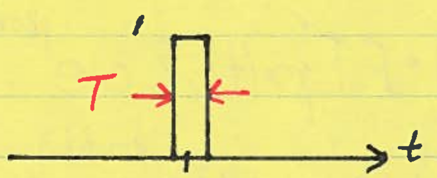




- how to deal with this?

- can still imagine the effects of an ideal pulse train $x(t)\,x_s(t)$

- but now the impulse response of my sampler is



therefore... $x_s(t) = p(t) * [x(t)\,x_s(t)]$

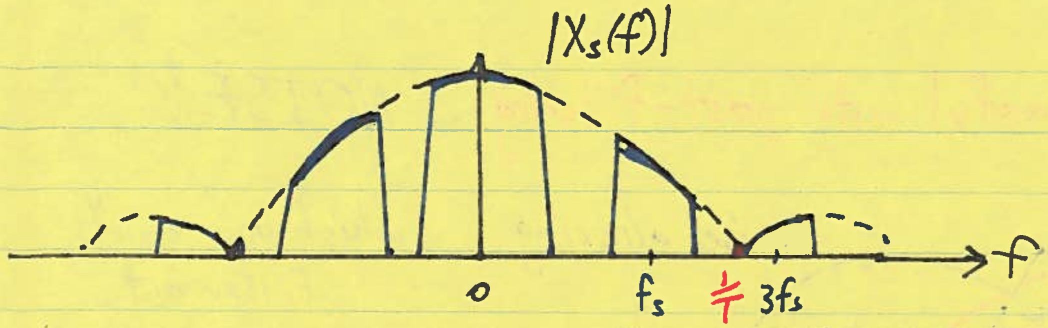
convolution

- and the frequency spectrum is

$$X_s(f) = P(f) \cdot \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad \& \quad P(f) = T \cdot \frac{\sin(\pi f T)}{\pi f T} = T\,\text{sinc}(fT)$$

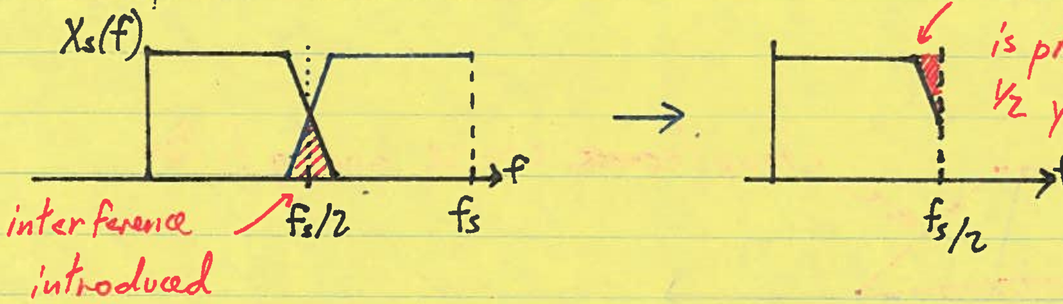
recall this from impulse sampling

so



5.6 Aliasing & Oversampling

- if you sample at too low a rate

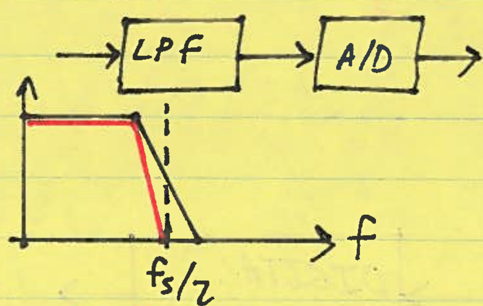


an interference is present up to 1/2 your sampling freq.

(imagine you filter out $f > \frac{f_s}{2}$

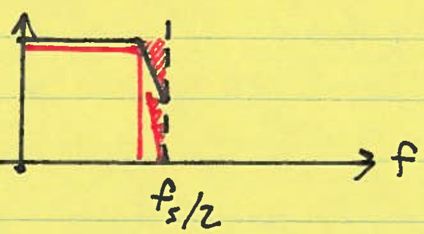
- solutions? (if you can't sample any higher)

- pre-filter to take care of distortion (assuming filtered signal is something you can live with



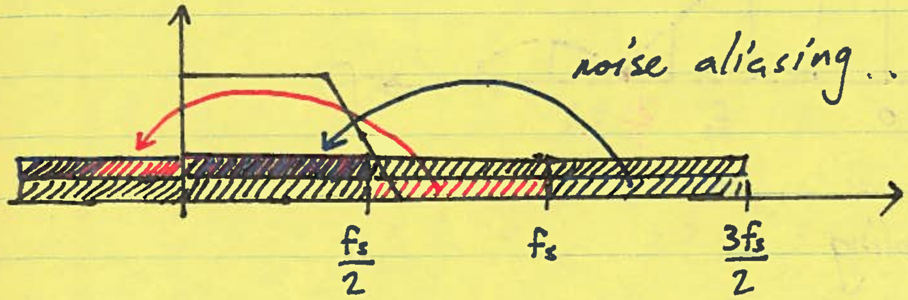
"anti-aliasing" filter

- post-filter



- may need more complex filtering for post
- BUT you can do it in DIGITAL

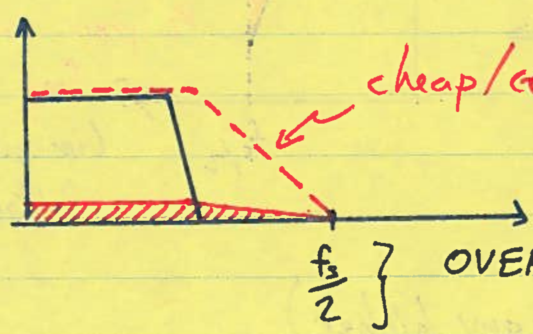
- be ~~cafe~~ careful with post-filtering... NOISE!



noise aliasing... which you can't filter out

- typically you do a bit of both

PRE:



cheap/~~coase~~ coarse analog filter

} OVERSAMPLE

sharp digital filter to get ~~did~~ of most of ~~reg~~ remaining noise