York University
Department of Electrical Engineering and Computer Science
EECS 4214

Lab #4 Matched Filtering and ISI

1 Purpose

In this lab, you will design optimum filters that maximize the output signal-to-noise ratio (SNR). Referred to as matched filters, such filters are used in many applications, including: (i) Digital communications to detect the presence of one of the two signals representing bits 0 and 1 for binary transmission; (ii) Radar, in which a known signal is sent out, and the signal reflected from the target (backscatter) is examined for unknown elements (range, velocity, and direction) attributed to the target, and; (iii) Image processing to improve SNR for medical images such as X-rays, MRIs, and CT scans. Though several different implementations of matched filters for different scenarios have been designed, you will consider the case of detecting a known, deterministic signal in additive white Gaussian noise. In particular, you will implement the correlator and time reversal implementations of the matched filter.

Another key issue is equalization, a technique used to reduce the effect of ISI. In this lab you will be calculating the necessary settings of some basic equalizer filters.

2 Objectives

By the end of this project, you will be able to:

1) Design the *correlator implementation* of the matched filter and test it in the MATLAB simulation environment to detect transmitted bits in a binary communication system.

2) Design the *time reversal implementation* of the matched filter and test it in the MATLAB simulation environment to detect transmitted bits in a binary communication system.

3) Compare the performance of the correlator and time-reversal implementations with each other.

4) Compute the coefficients of ZFE and MMS equalizer.

3 Reference

Bernard Sklar text: Sections 3.1 – 3.2. Pages 106 – 136.
Lecture 12 notes on equalization.

4 Binary Signal Detection in AWGN

In a binary communication system, binary data consisting of 0's and 1's are transmitted by means of two signal waveforms, say $s_0(t)$ and $s_1(t)$. Suppose that the data rate is specified at R bits per second. Then each bit is mapped into a corresponding signal waveform according to the rule

$$0 \longrightarrow s_0(t), \quad 0 \le t \le T \tag{1}$$

$$1 \longrightarrow s_1(t), \quad 0 \le t \le T \tag{2}$$

where $T = 1/R$ is defined as the bit time interval. We assume that the data bits 0 and 1 are equally probable, i.e., each occurs with a probability of 1/2. The channel through which the signal is transmitted is assumed to corrupt the signal by the addition of noise, denoted by $n(t)$, which is a sample function of white Gaussian noise with power spectrum $N_0/2$ Watts/Hz. Such a channel is called an additive white Gaussian noise (AWGN) channel. Consequently, the received waveform is expressed as

$$r(t) = s_i(t) + n(t), \quad i = 0, 1, \quad 0 \le t \le T. \tag{3}$$

The task of the receiver is to determine whether a 0 or a 1 was transmitted after observing the received signal $r(t)$ in the interval $0 \le t \le T$. The receiver is normally designed to minimize the probability of error. Such a receiver is based on the optimum principle of matched filters.

In this project, we will consider the optimum receivers for transmitting binary information through an additive white Gaussian noise (AWGN). The receivers being considered in this project are the signal correlator and the time reversal implementation of the matched filter. We provide a brief introduction to the two receivers without going into the derivations, which can be seen in the text.

5 Signal Correlator Implementation

The signal correlator cross-correlates the received signal $r(t)$ with the two possible transmitted signal $s_0(t)$ and $s_1(t)$. In other words the signal correlator computes the two outputs

$$r_0(t) = \int_0^t r(\tau) s_0(\tau) d\tau \tag{4}$$

$$r_1(t) = \int_0^t r(\tau) s_1(\tau) d\tau \tag{5}$$

within the interval $0 \le t \le T$, samples the two outputs at $t = T$, and feeds the sampled outputs to the detector. For equiprobable signals, the detector decides in favour of the signal with the higher correlator output. Thus $s_0(t)$ is selected if $r_0(t) > r_1(t)$, and vice versa.

6 Filter Implementation

An alternative approach to SNR optimization employs a filter with an impulse response, $h(t)$, matched to the signal waveform shape, $s(t)$, reversed in time

$$h(t) = s(T-t), \quad 0 \leq t \leq T. \tag{6}$$

as noted the response is time-limited to $0 \leq t \leq T$. Such a component is clearly a good motivation for the term "matched filter".

The output of the matched filter for the two possible signals is given by

$$y_0(t) = r(t) \otimes h_0(t) \tag{7}$$
$$y_1(t) = r(t) \otimes h_1(t) \tag{8}$$

where $\otimes$ is the convolution operator and $\{h_0(t), h_1(t)\}$ are the time-reversed impulse response implementations of the filter matched to $\{s_0(t), s_1(t)\}$ based on (6). The detector samples the two outputs $y_i(t)$ at $t = T$ and decides in favour of the signal that has a higher output. It may be noted that the matched filter output at the sampling instant $t = T$ is identical to the output of the signal correlator.

Problem 1 Suppose the two waveforms $s_0(t) = A\cos(\pi t/T)$ and $s_1(t) = A\cos(2\pi t/T)$ (each spanning $0 \leq t \leq T$) are used to transmit binary information through an AWGN channel. Attempting to correlate these two waveforms via the operation

$$\int_0^T s_0(t)s_1(t)\mathrm{d}t \tag{9}$$

results in 0 which means that the respective waveforms we are using to represent 0's and 1's are *orthogonal* to each other.

The received signal, $r(t)$, in each bit interval of duration T given by (3), is sampled at a rate of $10/T$, i.e., at ten samples per bit interval. Consequently, the sampled version of the received sequence when $s_m(t)$ is transmitted, is

$$r_k = s_{m,k} + n_k, \quad k = 1, \ldots, 10, \quad m \in \{0, 1\} \tag{10}$$

where the sequence $\{n_k\}$ is zero mean, Gaussian with each random variable having the variance σ^2 and where $s_{0,k}$ is the discrete-time version of $s_0(t)$ and $s_{1,k}$ is the discrete-time version of $s_1(t)$. Write a MATLAB routine that generates the sequence $\{r_k\}$ for each of the possible received signals, and perform a discrete-time correlation of the sequence $\{r_k\}$ with each of the possible signals $s_0(t)$ and $s_1(t)$ represented by their samples versions for different values of the AWGN variance $\sigma^2 = 0$, $\sigma^2 = 0.1$, $\sigma^2 = 1$, and $\sigma^2 = 2$. The signal amplitude may be normalized to $A = 1$. Plot the correlator outputs at time instant $k = 1, 2, 3, \ldots 10$. Comment on the results.

Problem 2 In this problem, the objective is to substitute two matched filters in place of the two correlators used in **Problem 1**. The condition for generating signals is identical to **Problem 1**.

Write a MATLAB function that generates the sequence $\{r_k\}$ for each of the two possible received signals, and perform the discrete-time matched filtering of the sequence $\{r_k\}$ with each of the two possible signals $s_0(t)$ and $s_1(t)$, represented by their sampled versions, for different values of the AWGN variance $\sigma^2 = 0$, $\sigma^2 = 0.1$, $\sigma^2 = 1$, and $\sigma^2 = 2$. Plot the correlator outputs at time instants corresponding to $k = 1, \ldots 10$. Comment on the results and compare to the correlator implementation.

Problem 3 Imagine that the impulse response from your matched filters is $h[k] = [36, 230, 97, 37, 18]$ mV. Calculate the normalized coefficients of a 4-tap ZFE-LS equalizer intended to result in an impulse response of $[0, 1, 0, 0]$ V. Repeat the exercise for a MMS equalizer if the SNR is only 5.