

1. (3 points) What is the average power of the signal $x(t) = 3 \cos(2\pi f_0 t + 33^\circ)$ [V/ $\sqrt{\text{Ohm}}$] ($f_0 = 1200$ Hz)? What is the average energy consumed over 5 ms (milliseconds). Clearly indicate units in your final answer.

$$P_x^T = \frac{3^2}{2} = 4.5 \text{ W}$$

$$\textcircled{B} P_x = \sum |c_n|^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 = \frac{3^2}{2}$$

$$E_x^T = P_x^T \cdot \text{time}$$

$$= 4.5 \times 5 \times 10^{-3}$$

$$= 22.5 \times 10^{-3} \text{ J}$$

2. (3 points) What is the Fourier transform, $X(f)$, of $x(t) = A \sin(\omega_0 t + \phi)$? Show your work.

$$\text{F.T. } \{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

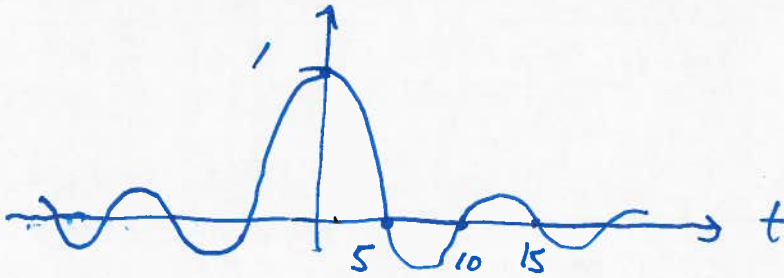
$$= \int_{-\infty}^{\infty} A \sin(\omega_0 t + \phi) e^{-2\pi f t} dt$$

$$= \frac{A}{2j} \int_{-\infty}^{\infty} [e^{j(\omega_0 t + \phi)} - e^{-j(\omega_0 t + \phi)}] e^{-2\pi f t} dt$$

$$= \frac{A}{2j} \times \text{F.T.} \{e^{j\omega_0 t}\} \times e^{j\phi} - \frac{A}{2j} \text{F.T.} \{e^{-j\omega_0 t}\} \times e^{-j\phi}$$

$$= \frac{A}{2j} \times e^{j\phi} \delta(f - f_0) - \frac{A}{2j} e^{-j\phi} \delta(f + f_0)$$

3. (1 point) Sketch the signal $x(t) = \text{sinc}(t/5)$.



4. (3 points) Sketch the spectrum $X(f)$ of $x(t) = \text{sinc}(t/5) \cdot \cos(2\pi 500t)$.



$c = 3 \times 10^8$ m/s (in free space), $c = 2 \times 10^8$ m/s (in media), 1 km = 10^3 m, 1 ms = 10^{-3} s, 1 Mb = 10^6 b

$$\mathcal{F}\{\text{rect}(t/T)\} = T \text{sinc}(fT) = T \sin(\pi fT) / \pi fT$$

$$\mathcal{F}\{\text{sinc}(t/T)\} = \frac{1}{T} \text{rect}(fT), \mathcal{F}\{e^{j\omega_0 t}\} = \delta(f - f_0)$$

$$\mathcal{F}\{1 - |\tau|/T\} = T \text{sinc}^2(fT), X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\Omega}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b, \cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b, \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\cos^2 a = 0.5(1 + \cos 2a), \sin 2a = 2 \sin a \cos a, \cos 2a = \cos^2 a - \sin^2 a = 2 \cos^2 a - 1$$

$$\cos a = (e^{ja} + e^{-ja})/2, \sin a = (e^{ja} - e^{-ja})/j2, \tan a = \sin a / \cos a$$

$$\psi_x(f) = |X(f)|^2, G_x(f) = \sum |c_n|^2 \delta(f - nf_0), G_x(f) = \lim_{T \rightarrow \infty} |X_T(f)|^2$$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt, R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} x(t)x(t+\tau)dt$$