Counterexamples and Witnesses

EECS 4315

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The Simpsons

Since its debut on December 17, 1989, the show has broadcast 590 episodes and the twenty-seventh season started airing on September 27, 2015. The Simpsons is the longest-running American sitcom, the longest-running American animated program, and the longest-running American prime-time, scripted television series.

The Simpsons has won dozens of awards since it debuted as a series, including 31 Prime-time Emmy Awards, 30 Annie Awards and a Peabody Award.

Source: FoxFlash

Gone, Maggie, Gone

Gone, Maggie, Gone is the thirteenth episode of The Simpsons' twentieth season. It originally aired on March 15, 2009. In the episode, Homer leaves Maggie on the doorstep of a convent, but when she disappears, Lisa goes undercover as a nun to solve the mystery. Meanwhile, Homer tries to keep Maggie's disappearance a secret from Marge, who was temporarily blinded while watching a solar eclipse.

Source: FoxFlash

Maggie-Dog-Poison problem

Let us model Homer, Maggie, Santa's Little Helper and the poison crossing by boat model as a transition system.

Question

How can we represent the states of the transition system?

Maggie-Dog-Poison problem

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Question

How can we represent the states of the transition system?

Question

How is the transition relation defined?

Maggie-Dog-Poison problem

Let us model Homer, Maggie, Santa's Little Helper and the poison crossing by boat model as a transition system.

Question

How can we represent the states of the transition system?

Question

How is the transition relation defined?

Question

What is the initial state?

Maggie-Dog-Poison problem

Let us express properties of this model, such as “whenever Maggie and the poison are on the same side, then Homer is there as well.”

Question

Which labels do we need to express such properties?

Maggie-Dog-Poison problem

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Question

Which labels do we need to express such properties?

Question

How is the labelling function defined?

Maggie-Dog-Poison problem

Question

How do we express in CTL “whenever Maggie and the poison are on the same side, then Homer is there as well?”

Maggie-Dog-Poison problem

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How do we express in CTL “whenever Maggie and the poison are on the same side, then Homer is there as well?”

Question

How do we express the Maggie-Dog-Poison problem in CTL?

Maggie-Dog-Poison problem

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How do we express in CTL “whenever Maggie and the poison are on the same side, then Homer is there as well?”

Question

How do we express the Maggie-Dog-Poison problem in CTL?

Question

Does the model satisfy this property?

Maggie-Dog-Poison problem

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How do we express in CTL “whenever Maggie and the poison are on the same side, then Homer is there as well?”

Question

How do we express the Maggie-Dog-Poison problem in CTL?

Question

Does the model satisfy this property?

Question

So there exists a path, but which one?

Witness

Definition

A *witness* of a CTL formula $\exists\varphi$ is a sufficiently long prefix of a path π with $\pi \models \varphi$.

Witness

Question

What is a witness of $\exists \bigcirc \Phi$?

Witness

Question

What is a witness of $\exists \bigcirc \Phi$?

Answer

An initial path fragment $s_0 s_1$ such that $s_1 \models \Phi$.

Witness

Question

What is a witness of $\exists(\Phi \, U \, \Psi)$?

Witness

Question

What is a witness of $\exists(\Phi \, \mathrm{U} \, \Psi)$?

Answer

An initial path fragment $s_0 \ldots s_n$ for some $n \geq 0$ such that $s_n \models \Psi$ and $s_i \models \Phi$ for all $0 \leq i < n$.

Witness

Question

What is a witness of $\exists \Box \Phi$?

Witness

Question

What is a witness of $\exists\Box\Phi$?

Answer

An initial path fragment $s_0 \ldots s_m \ldots s_n$ for some $m \geq 0$ and $n > m$ such that $s_i \models \Phi$ for all $0 \leq i < n$ and $s_m = s_n$.

Maggie-Dog-Poison problem

Question

How do we express in CTL “No matter how Homer acts, Maggie is never alone?”

Maggie-Dog-Poison problem

Question

How do we express in CTL “No matter how Homer acts, Maggie is never alone?”

Question

Does the model satisfy this property?

Maggie-Dog-Poison problem

Question

How do we express in CTL “No matter how Homer acts, Maggie is never alone?”

Question

Does the model satisfy this property?

Question

So there exists a path that does not satisfy the property, but which one?

Counterexample

Definition

A *counterexample* of a CTL formula $\forall\varphi$ is a sufficiently long prefix of a path π with $\pi \not\models \varphi$.

Counterexample

Question

What is a counterexample of $\forall \bigcirc \Phi$?

Counterexample

Question

What is a counterexample of $\forall \bigcirc \Phi$?

Answer

An initial path fragment $s_0 s_1$ such that $s_1 \not\models \Phi$.

Counterexample

Question

What is a counterexample of $\forall(\Phi \, U \, \Psi)$?

Counterexample

Question

What is a counterexample of $\forall(\Phi \, U \, \Psi)$?

Answer

An initial path fragment

- $s_0 \ldots s_n$ for some $n \geq 0$ such that $s_n \models \neg\Phi \wedge \neg\Psi$ and $s_i \models \Phi \wedge \neg\Psi$ for all $0 \leq i < n$, or
- $s_0 \ldots s_m \ldots s_n$ for some $m \geq 0$ and $n > m$ such that $s_i \models \Phi \wedge \neg\Psi$ for all $0 \leq i < n$ and $s_m = s_n$.

Counterexample

Question

What is a counterexample of $\forall\Box\Phi$?

Counterexample

Question

What is a counterexample of $\forall\Box\Phi$?

Answer

An initial path fragment $s_0 \ldots s_n$ for some $n \geq 0$ such that $s_i \models \Phi$ for all $0 \leq i < n$ and $s_n \not\models \Phi$.

Quiz on Thursday

- Quiz will cover the reading material of March 8 and 14.
- Quiz will be 30 minutes.
- Definitions will be provided.