

## Arithmetic for Computers

Operations on integers

- Addition and subtraction
- Multiplication and division
- Dealing with overflow

Floating-point real numbers

- Representation and operations


## Integer Addition

Example: $7+6$

|  | (0) |  | (0) |  | (1) |  | (1) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 |  | 0 |  | 0 |  | 1 |  | 1 |  |  |  |
|  | 0 |  | 0 |  | 0 |  | 1 |  | 1 |  |  |  |
| . . (0) | 0 | (0) | 0 | (0) | 1 | (1) | 1 | 1) | 0 |  |  |  |

## Overflow if result out of range

- Adding +ve and -ve operands, no overflow
- Adding two +ve operands

Overflow if result sign is 1

- Adding two -ve operands

Overflow if result sign is 0

## Integer Subtraction

Add negation of second operand

| Example: $7-6=7+(-6)$ | 111111001 |
| :---: | ---: |
| $+7:$ | $00000000 \ldots 00000111$ |
| $\frac{1111111010}{110}$ |  |

-6: $11111111 \ldots 11111010$
+1: $00000000 \ldots 00000001$

## Overflow if result out of range

- Subtracting two +ve or two -ve operands, no overflow
- Subtracting +ve from -ve operand

Overflow if result sign is 0

- Subtracting -ve from +ve operand

Overflow if result sign is 1

## Dealing with Overflow

Some languages (e.g., C) ignore overflow - Use MIPS addu , addui , subu instructions Other languages (e.g., Ada, Fortran) require raising an exception

- Use MIPS add, addi, sub instructions
- On overflow, invoke exception handler

Save PC in exception program counter (EPC) register
Jump to predefined handler address
mf c 0 (move from coprocessor reg) instruction can retrieve EPC value, to return after corrective action

## Arithmetic for Multimedia

Graphics and media processing operates on vectors of 8-bit and 16-bit data

- Use 64-bit adder, with partitioned carry chain

Operate on $8 \times 8$-bit, $4 \times 16$-bit, or $2 \times 32$-bit vectors

- SIMD (single-instruction, multiple-data)

Saturating operations

- On overflow, result is largest representable value
c.f. 2s-complement modulo arithmetic
- E.g., clipping in audio, saturation in video



## Optimized Multiplier

## Perform steps in parallel: add/shift



- One cycle per partial-product addition
- That's ok, if frequency of multiplications is low

Chapter 3 - Arithmetic for Computers - 9

## Faster Multiplier

## Uses multiple adders

- Cost/performance tradeoff


Can be pipelined

- Several multiplication performed in parallel


## MIPS Multiplication

Two 32-bit registers for product

- HI: most-significant 32 bits
- LO: least-significant 32-bits


## Instructions

- mult rs, rt / multurs, rt

64-bit product in $\mathrm{HI} / \mathrm{LO}$

- mf hi rd / mflord

Move from $\mathrm{HI} / \mathrm{LO}$ to rd
Can test HI value to see if product overflows 32 bits

- mul rd, rs, rt

Least-significant 32 bits of product $\rightarrow$ rd

## Division

| quotient | Check for 0 divisor <br> Long division approach |
| :---: | :---: |
| dividend If divisor $\leq$ dividend bits |  |
| 1 bit in quotient, subtract |  |

## Division Hardware



## Optimized Divider



One cycle per partial-remainder subtraction
Looks a lot like a multiplier!

- Same hardware can be used for both



## Faster Division

Can't use parallel hardware as in multiplier

- Subtraction is conditional on sign of remainder Faster dividers (e.g. SRT devision) generate multiple quotient bits per step
- Still require multiple steps


## MIPS Division

Use HI/LO registers for result

- HI: 32-bit remainder
- LO: 32-bit quotient


## Instructions

- div rs, rt / divurs, rt
- No overflow or divide-by-0 checking

Software must perform checks if required

- Use mf hi , mf Io to access result

