

## Chapter Roadmap

How to design an ALU?
MIPS datapath
Pipelining
Hazards
Real stuff

## Introduction

## CPU performance factors

- Instruction count

Determined by ISA and compiler

- CPI and Cycle time

Determined by CPU hardware
We will examine two MIPS implementations

- A simplified version
- A more realistic pipelined version

Simple subset, shows most aspects

- Memory reference: I w, sw
- Arithmetic/logical: add, sub, and, or, sl t
- Control transfer: beq, j



## Possible Representations

| Sign Mag. | Two's Comp. | One's Comp. |
| :---: | :---: | :---: |
|  | $1000=-8$ |  |
| $1111=-7$ | $1001=-7$ | $1000=-7$ |
| $1110=-6$ | $1010=-6$ | $1001=-6$ |
| $1101=-5$ | $1011=-5$ | $1010=-5$ |
| $1100=-4$ | $1100=-4$ | $1011=-4$ |
| $1011=-3$ | $1101=-3$ | $1100=-3$ |
| $1010=-2$ | $1110=-2$ | $1101=-2$ |
| $1001=-1$ | $1111=-1$ | $1110=-1$ |
| $1000=-0$ |  | $1111=-0$ |
| $0000=+0$ | $0000=0$ | $0000=+0$ |
| $0001=+1$ | $0001=+1$ | $0001=+1$ |
| $0010=+2$ | $0010=+2$ | $0010=+2$ |
| $0011=+3$ | $0011=+3$ | $0011=+3$ |
| $0100=+4$ | $0100=+4$ | $0100=+4$ |
| $0101=+5$ | $0101=+5$ | $0101=+5$ |
| $0110=+6$ | $0110=+6$ | $0110=+6$ |
| $0111=+7$ | $0111=+7$ | $0111=+7$ |

Issues:

- balance
- number of zeros
- ease of operations

Which one is best? Why?

## MIPS Representations

32 -bit signed numbers (2's complement):

```
00000000 0000 0000 0000 0000 0000 0000 two = 0 0 ten
00000000 0000 0000 0000 0000 0000 0001 two = + 1 (ten
00000000 0000 0000 0000 0000 0000 0010 two }=+\mp@subsup{2}{\mathrm{ ten m maxint}}{\mathrm{ ten m}
...
```

```
0111 1111 1111 1111 1111 1111 1111 1110 
```

0111 1111 1111 1111 1111 1111 1111 1110
0111 1111 1111 1111 1111 1111 1111 1111 two = + 2,147,483,647 ten

```
0111 1111 1111 1111 1111 1111 1111 1111 two = + 2,147,483,647 ten
```






```
1000 0000 0000 0000 0000 0000 0000 0010 two = - 2,147,483,646 ten
```

1000 0000 0000 0000 0000 0000 0000 0010 two = - 2,147,483,646 ten
...

```
...
```

$1111111111111111111111111111{1101_{\text {two }}=-3}^{\text {ten }}$
$1111111111111111111111111111{1110_{\mathrm{two}}}^{1} 1112-2_{\text {ten }}$
$1111111111111111111111111111{1111_{\mathrm{two}}=-1_{\text {ten }}, ~}_{111}$

What if the bit string represented addresses?
need operations that also deal with only positive (unsigned)
integers

## Two's Complement Operations

Negating a two's complement number - complement all the bits and then add a 1

- remember: "negate" and "invert" are quite different!
- Starting from LSb, all 0's as is, first 1 as is, then invert

Converting n-bit numbers into numbers with more than n bits:

- MIPS 16-bit immediate gets converted to 32 bits for arithmetic
- sign extend - copy the most significant bit (the sign bit) into the other bits

$$
\begin{array}{llll}
0010 & -> & 0000 & 0010 \\
1010 & -> & 1111 & 1010
\end{array}
$$

- sign extension versus zero extend (lb vs. lbu)


## Addition \& Subtraction

Just like in grade school (carry/borrow 1s)

| 0111 |  |  |
| ---: | ---: | ---: |
| $+\quad 0110$ |  |  |
| 1101 | $-\quad 01110$ | 0110 |
| 0001 | $-\quad 0101$ |  |
| 0001 |  |  |

Two's complement operations are easy

- do subtraction by negating and then adding

| 0111 | $\rightarrow$ |
| ---: | :--- |
| -0110 | $\rightarrow$ |
| 0001 |  |

Overflow (result too large for finite computer word)

- e.g., adding two n-bit numbers does not yield an n-bit number

0111
$\begin{array}{r}+\quad 0001 \\ \hline\end{array}$
1000

## Building a 1-bit Binary Adder



| A | B | carry_in | carry_out | S |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

> S = A xor B xor carry_in
> carry_out =A\&B | A\&carry_in | B\&carry_in
(majority function)
$\square$ How can we use it to build a 32-bit adder?
How can we modify it easily to build an adder/subtractor?

## Building 32-bit Adder


$\square$ Just connect the carry-out of the least significant bit FA to the carry-in of the next least significant bit and connect . . .

## $\square$ Ripple Carry Adder (RCA)

- advantage: simple logic, so small (low cost)
- disadvantage: slow and lots of glitching (so lots of energy consumption)

A 32-bit Ripple Carry Adder/Subtractor


## Overflow Detection and Effects

Overflow: the result is too large to represent in the number of bits allocated
When adding operands with different signs, overflow cannot occur! Overflow occurs when

- adding two positives yields a negative
- or, adding two negatives gives a positive
- or, subtract a negative from a positive gives a negative
- or, subtract a positive from a negative gives a positive
- On overflow, an exception (interrupt) occurs
- Control jumps to predefined address for exception
- Interrupted address (address of instruction causing the overflow) is saved for possible resumption
Don't always want to detect (interrupt on) overflow

A Simple ALU Cell with Logic Op Support




## Modifying the ALU Cell for slt



## Overflow Detection

Overflow occurs when the result is too large to represent in the number of bits allocated

- adding two positives yields a negative
- or, adding two negatives gives a positive
- or, subtract a negative from a positive gives a negative
- or, subtract a positive from a negative gives a positive
- On your own: Prove you can detect overflow by:
- Carry into MSB xor Carry out of MSB

| 0 | 1 | 1 | 1 | 7 |
| ---: | ---: | ---: | ---: | ---: |
| + | 0 | 0 | 1 | 1 |


| 1 | 1 | 0 | 0 | -4 |
| :---: | :---: | :---: | :---: | :---: |
| + | 1 | 0 | 1 | 1 |



