Laboratory 3 – Fourier series, Fourier transform and Bode Plots in MATLAB

Introduction:

The objectives of this lab are to use MATLAB:

1. To plot periodic signals with Fourier series representation
2. To obtain and plot the output response signal with periodic input signal
3. To learn and create plots for the frequency response of a system (i.e. Bode plot) in MATALB

There are **5** exercises in this lab.

Note: You are encouraged to complete the pre-lab exercises before you begin this lab.

Periodic signals with Fourier series representation

Recall that an arbitrary periodic signal $x(t)$ with a fundamental period T_0 can be expressed as an exponential Fourier series as follows:

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

where the exponential Fourier series coefficients D_n are calculated as: $D_n = \dfrac{1}{T} \int_{\langle T_0 \rangle} x(t) e^{-jn\omega_0 t} dt$

In an LTIC system with impulse response $h(t)$ as shown in Fig. 1, the output response signal is then given by:

$$y(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} H(\omega)\Big|_{\omega=n\omega_0} \quad \text{where } H(\omega) \text{ is the Fourier transform of } h(t)$$

Hence, the output signal $y(t)$ is another periodic signal with the same fundamental period T_0.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Fig. 1

Laboratory exercise 1.

a) Consider the following periodic signal $x(t)$, which is a full-wave rectified sine-wave. Using the Fourier coefficients that you determined from the pre-lab exercise, plot $x(t)$ in MATALB that considers the first 40 terms in the Fourier series if $T = 2$ seconds. Plot $x(t)$ for $-4 \le t \le 4$. [Note: you should not simply enter term by term in MATLAB to complete the plot]
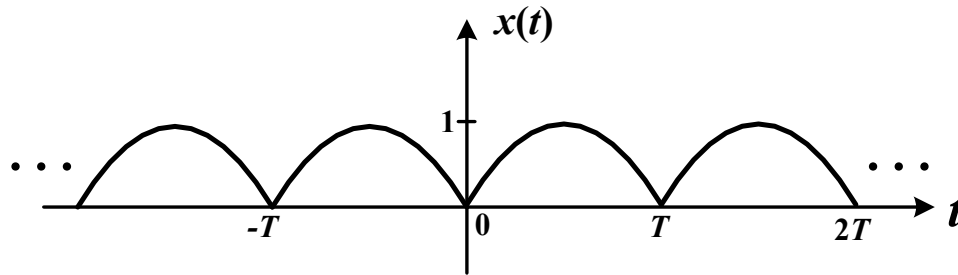
b) Based on the Fourier coefficients that you determined from the pre-lab exercise, if $T = 2$ seconds, plot both the magnitude spectrum and phase spectrum of $x(t)$ as a function of n for $-10 \leq n \leq 10$ in MATLAB. Remember the magnitude spectrum is the magnitude of the exponential Fourier series coefficients (i.e. D_n) and the phase spectrum is the phase of the exponential Fourier series coefficients. Use MATLAB built-in function `stem()` to plot all the points in both plots. Use `subplot()` to plot the magnitude spectrum in the upper plot and the phase spectrum in the lower plot. Label both plots.

c) $x(t)$ is now an input signal for an LTIC system with impulse response $h(t)$, and the Fourier transform of $h(t)$ in this system is: $H(\omega) = \dfrac{10}{30.5 + j\omega}$. Determine the output response signal $y(t)$. Plot both $x(t)$ and the output signal $y(t)$ on the same graph. Plot a few cycles. [Note: you need to complete all the mathematical analysis in MATLAB].

Laboratory exercise 2.

a) Consider a symmetrical square wave signal $x_2(t)$ with an amplitude of 1 and a fundamental period of T_0, the Fourier series is given as:

$$x_2(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4}{\pi n} \sin(n\omega_0 t)$$

Determine the exponential Fourier series of this signal and plot it in MATLAB for $-25 \leq n \leq 25$.
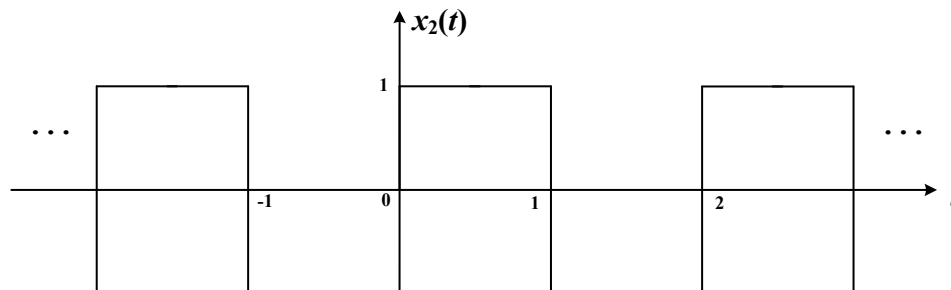
b) If an LTIC system with impulse response $h_2(t)$ and the Fourier transform of $h_2(t)$ is given as:

$H_2(\omega) = \dfrac{8}{20 + j\omega}$, determine the output response signal $y_2(t)$. Plot both $x_2(t)$ and the output signal $y(t)$ on the same graph for $-25 \leq n \leq 25$.

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c) Suppose the impulse response $h_2(t)$ is modified and is given as: $H_2(\omega) = \dfrac{8}{2 + j\omega}$. Plot both $x_2(t)$

and the output signal $y_2(t)$ on the same graph for $-25 \leq n \leq 25$. Comment on the differences that you observe in the new output response signal.

Creating Bode plots (i.e. gain and phase responses) in MATLAB

The Fourier complex expression (we usually call this the "transfer function") $H(\omega)$ provides a complete description of the LTIC systems. In many applications, the graphical plots of and versus frequency (ω) are used to analyze the characteristics of the LTIC system.

The magnitude spectrum response function is also referred to as the gain response of the system, while the phase spectrum is referred to as the phase response of the system. Here, we introduce Bode plots, where a logarithmic scale is used for the frequency ω –axis for both gain and phase responses of the system.

In Bode plots, the magnitude $|H(\omega)|$ in decibels (dBs) and phase $\angle H(\omega)$ are plotted as functions of frequency ω using a logarithmic scale. Use of a logarithmic scale, with base 10, on the frequency ω –axis has the advantage of allowing a wider range of frequencies to be plotted, with the lower frequencies represented at a higher resolution.

Hence, in a Bode plot, the magnitude $|H(\omega)|$ is expressed in dBs as $20\log_{10}|H(\omega)|$ and is plotted as a function of $\log_{10}\omega$. The phase Bode plot then plots the phase $\angle H(\omega)$ as a function of $\log_{10}\omega$. We will spend more time in the lectures to discuss how to sketch and create a Bode plot.

In MATLAB, there is a **bode** function that is used to sketch the Bode plot. Suppose we want to create the Bode plot of the following Fourier transfer function $H(\omega)$:

$$H(\omega) = \frac{1}{0.2 - 3j\omega} \tag{1}$$

The above Fourier transfer function is expressed in MATLAB in terms of the Laplace variable $s = j\omega$. In the next chapter, we will show that the independent variable s represents the entire complex plane and leads to the generalization of the Fourier transfer function into an alternative transfer function, known as Laplace transfer function. Hence, substituting into (1) results in:

$$H(s) = \frac{1}{0.2 - 3s} \tag{2}$$

Given $H(s)$, the Bode plots are obtained in MATLAB using the following instructions:

```
num_coefficient = 1;                        % coefficients of the numerator
den_coefficient = [-3 0.2];                 % coefficients of the denominator
system = tf(num_coefficient, den_coefficient)% specify the transfer function
bode(system, {0.001, 100});                 % sketch the Bode plot of the
transfer function
grid;
```

In the above MATLAB instructions, we have used 2 new built-in functions: **tf()** and **bode()**. The function **tf()** specifies the LTIC system $H(s)$ in terms of the coefficients of the polynomials of s in the numerator and denominator. The built-in function **bode()** sketches the Bode plots. It accepts 2 input arguments. The first input argument sys is used to represent the LTIC system, while the second input argument {0.001, 100} specifies the frequency range, 0.01radians/sec to 1000 radians/sec, used to sketch the Bode plots.
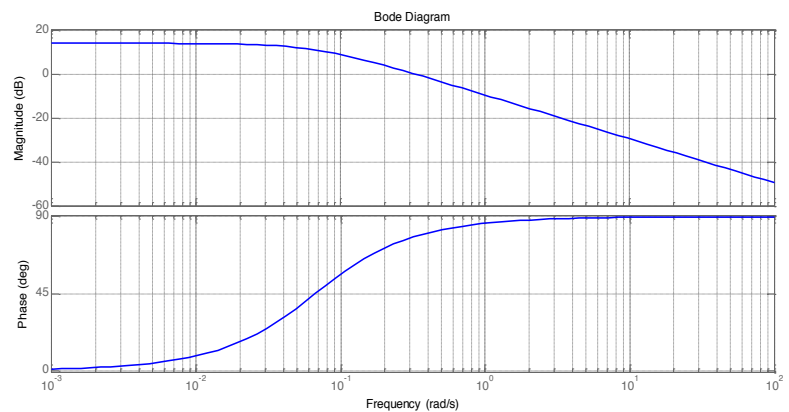
When the above instructions are executed in MATLAB, you will see the following from the MATLAB Command window, as well as the Bode plots of `system`.

```
system =

     -1
  ---------
  3 s - 0.2

Continuous-time            transfer
function.
```


Note that the second input argument in **bode()** can be omitted and the **bode** instruction will return a Bode plot with automatically adjusted frequency range determined by MATLAB. You can also plot multiple Bode plots on the same graph using the following structure:

```
bode(system1, system2, …, system_n);
```

Laboratory exercise 3.

a) Consider the two impulse response functions again in exercise 2b) and 2c), give the Bode plots of both functions and plot them on the same graph. Label your plots.

b) Now, with the help of the Bode plots, explain the differences that you observed in $y(t)$ in exercises 2b) and 2c).

c) Plot the output response signal $y(t)$ if the input signal is given by the square wave signal in exercise 2, and the impulse response is given by the cascaded connection of $h_{2b}(t)$ and $h_{2c}(t)$, that is:
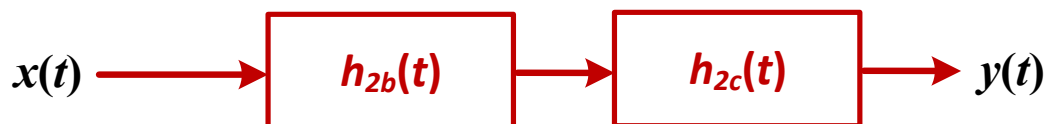
where $H_{2b}(\omega) = \dfrac{8}{20 + j\omega}$ and $H_{2b}(\omega) = \dfrac{8}{2 + j\omega}$

Laboratory exercise 4.

Consider the circuit in Fig. 2, which is an LTIC system that can be represented by a 1st order differential equation relating $x(t)$ and $y(t)$. If $R_1 = 10$, $R_L = 100$, $C_1 = 100\mu F$, based on the Fourier transfer function that you determined from the pre-lab exercise, obtain the Bode plots of this LTIC system (i.e. $y(s)/x(s)$) from MATLAB.
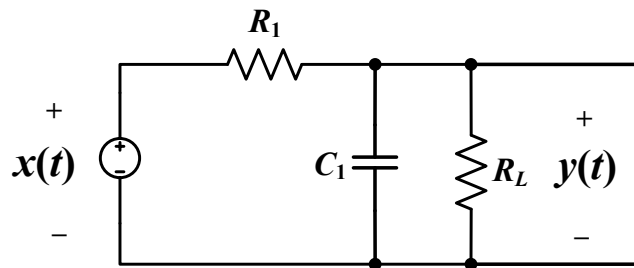

Fig. 2

Laboratory exercise 5.

a) Consider the circuit in Fig. 3, which is an LTIC system that can be represented by a 2nd order differential equation relating $x(t)$ and $y(t)$. If $R_L = 2\Omega$, $L = 1mH$, $C = 10\mu F$, based on the Fourier transfer function that you determined from the pre-lab exercise, obtain the Bode plots of this LTIC system (i.e. $y(s)/x(s)$) from MATLAB.

b) Now R_L is increased to 200Ω, with L and C remain the same, obtain the Bode plots of this LTIC system again from MATLAB. What did you observe? Explain.
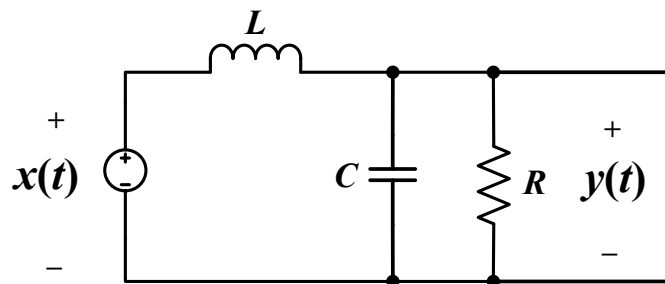

Fig. 3

Submitting your M-files and published report:

You can submit your files and report to eecs2602@gmail.com or hardcopy report.

Files to be submitted:

1. Main M-file: the main file should contain your solutions in exercise 1 – 5.
2. Published report of the main M-file in **pdf** (in your M-file, select the "PUBLISH" Tab

    and click)