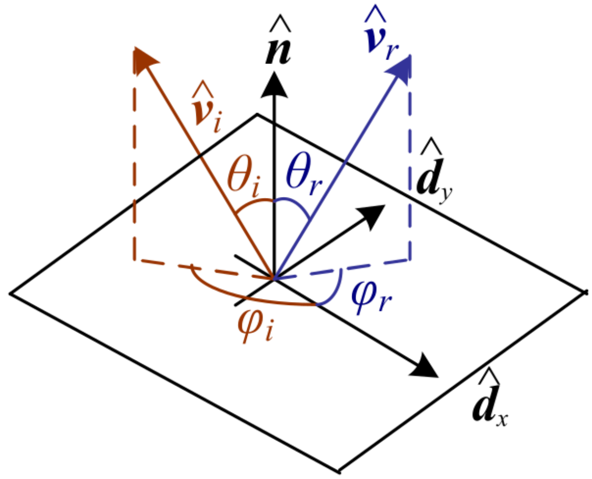
EECS 4422/5323 Assignment 1 Written Question Solutions

Question 1: BRDFs

Note that for all of these equations, it is important to think about the range of values that the angles can take. As shown in the diagram reproduced here on the right, azimuth angles ϕ_i and ϕ_r are taken from the range $[0, 2\pi]$, while the zenith angles θ_i and θ_r are taken from the range $[0, \frac{\pi}{2}]$.

For all parts to this question, the BRDF must have three properties to be considered realistic:

1. positive over all possible values
2. Helmholtz reciprocity
3. Energy conservation (the integral over outgoing rays must be less than or equal to 1)



BRDF angles diagram

Note that this third characteristic should involve a $\cos(\theta_r)$ term in the integral, but for ease of computation we can neglect this term for (b.) by noting that $f_r \cos(\theta_r) \leq f_r$. Thus, if $\int_0^{2\pi} \int_0^{\frac{pi}{2}} f_r \mathrm{d}\theta_r \mathrm{d}\phi_r \leq 1$, we can conclude that $\int_0^{2\pi} \int_0^{\frac{pi}{2}} f_r \cos(\theta_r) \mathrm{d}\theta_r \mathrm{d}\phi_r \leq 1$.

[7] marks each.

(a.)

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{1}{\pi} \sin(\theta_r)$$

Solution:

1. $\sin(\theta) > 0 for \theta \in [0, \frac{\pi}{2}]$, so $f_r > 0$, so this BRDF passes the positivity check
2. This BRDF violates the principle of Helmholtz reciprocity (counter-example: $f_r(\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{4}, \pi) = \sin(\frac{\pi}{4}) \neq f_r(\frac{\pi}{4}, \pi, \frac{\pi}{3}, \frac{\pi}{2}) = \sin(\frac{\pi}{3}))$
3. This BRDF actually (barely) passes the energy conservation requirement, $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin(\theta_r) \cos(\theta_r) \mathrm{d}\theta_r \mathrm{d}\phi_r = 1$

(b.)

$$f_r(\theta_i,\phi_i,\theta_r,\phi_r)=\begin{cases}\frac{0.1}{\pi}+0.2(\cos(\phi_i+\pi-\phi_r)\cos(\theta_r-\theta_i)) & |\phi_i+\pi-\phi_r|<0.1 \wedge |\theta_r-\theta_i|<0.1\\ \frac{0.1}{\pi} & \text{otherwise}\end{cases}$$

Solution:

1. This BRDF passes the positivity check (shown below)
2. This BRDF fails the reciprocity check (shown below)
3. This BRDF passes the energy conservation requirement (shown below)

Positivity: To show this, we need to show that $f_r > 0$ for all angle values. It is easy to show that when $|\phi_i+\pi-\phi_r| \geq 0.1$ or $|\theta_r-\theta_i| \geq 0.1$ this condition is satisfied, since f_r is constant and > 0. If we do the substitutions $x=\phi_r+\pi-\phi_i$, $y=\theta_r-\theta_i$, we can see that $\cos(x) > 0$ for $-0.1 < x < 0.1$ (and likewise for $\cos(y)$ over the same range of values). Thus, $\frac{0.1}{\pi}+0.2(\cos(x)\cos(y)) > 0$ for $|x| < 0.1, |y| < 0.1$, and therefore $f_r > 0$ always.

Reciprocity failure: This can be shown either with an example or algebraically. Algebraically, we can see that this does not work in general by noting that $-(\phi_i+\pi-\phi_r) = (\phi_r-\pi-\phi_i) \neq (\phi_r+\pi-\phi_i)$. This means that there are some angle values such that $|\phi_i+\pi-\phi_r| < 0.1$, but $|\phi_r+\pi-\phi_i| \geq 0.1$.

Thus, for example, take $\theta_i=\theta_r, \phi_i=\frac{\pi}{2}, \phi_r=\frac{3\pi}{2}$. Then

$|\phi_i+\pi-\phi_r| = |\frac{\pi}{2}+\pi-\frac{3\pi}{2}| = 0 < 0.1$

but

$|\phi_r+\pi-\phi_i| = |\frac{3\pi}{2}+\pi-\frac{\pi}{2}| = 2\pi > 0.1$

Therefore for this case, $f_r(\theta_i,\phi_i,\theta_r,\phi_r) \neq f_r(\theta_r,\phi_r,\theta_i,\phi_i)$.

Energy conservation: We can see this by applying the sum rule and the substitutions $x=\phi_r+\pi-\phi_i$, $y=\theta_r-\theta_i$:

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\frac{0.1}{\pi}\mathrm{d}\theta_r\mathrm{d}\phi_r+\int_{-0.1}^{0.1}\int_{-0.1}^{0.1}0.2(\cos(x)\cos(y))\mathrm{d}x\mathrm{d}y \approx 0.314+0.008=0.322$$

Since this value is an upper bound on the integral over the BRDF and is well less than 1, we can conclude that this BRDF satisfies energy conservation.

(c.)

$$f_r(\theta_i,\phi_i,\theta_r,\phi_r) = \begin{cases} \frac{0.2}{\pi} + 0.1(\cos(\phi_i+\pi-\phi_r) + 0.2\cos(\theta_r-\theta_i)) & \phi_i < \pi \wedge |\theta_r-\theta_i| < 0.1 \\ \frac{0.3}{\pi} & \text{otherwise} \end{cases}$$

1. This BRDF fails the positivity check (shown below)
2. This BRDF fails the reciprocity check (shown below)
3. Given the failure of the previous two conditions, it should not be necessary to check this condition. Should you wish to check it, though, the procedure would be similar to *(b.)* above, just messier and with more cases.

Positivity failure: We can show that this BRDF is sometimes negative by noting that the condition for the more complicated expression holds so long as $\phi_i < \pi$, but does not depend on the value of ϕ_r. Thus, when $\phi_i = \phi_r, \phi_i < \pi, |\theta_r - \theta_i| < 0.1$, we have the following:

$$\frac{0.2}{\pi}+0.1(\cos(\pi)+0.2\cos(\theta_r-\theta_i)) \leq \frac{0.2}{\pi}+0.1(\cos(\pi)+0.2\cos(0) = \frac{0.2}{\pi}+0.1(-1+0.2) \approx 0.064-0.08 < 0$$

Reciprocity failure: Assume $|\theta_r - \theta_i| < 0.1, \phi_i < \pi, \phi_r > \pi$, then:

$$f_r(\theta_i,\phi_i,\theta_r,\phi_r) = \frac{0.2}{\pi} + 0.1(\cos(\phi_i+\pi-\phi_r) + 0.2\cos(\theta_r-\theta_i)) \neq f_r(\theta_r,\phi_r,\theta_i,\phi_i) = \frac{0.3}{\pi}$$

Question 2: Propeller Image

(a.) [9 points] Write a brief explanation for the strange appearance of the aircraft propeller, including what specific aspect of image capture is responsible.

Answer: The propeller is spinning fast enough that its rate of displacement is at a similar speed to the speed of aquisition for a rolling shutter camera, and therefore different pixel rows are actually displaced slightly in time. For slower moving objects this would not be noticable, but for the propeller it results in each row of pixels corresponding to a different rotational position of the propeller, leading to the odd spatial displacements in the image.

(b.) [4 points] What can you likely conclude about the camera Fernanda was using when she took this photo?

Answer: Fernanda's camera likely has a CMOS sensor, as CMOS sensors typically utilize rolling shutters.

(c.) [6 points] Fernanda says that in person the propeller looked to her like a "ghostly circle" (*i.e.* a uniform semi-transparent circle). Can you briefly explain why it would appear that way to a human observer?

Answer: Unlike a camera with a synchronous frame acquisition, the human eye samples light continuously and asynchronously. For all points along the path of the propeller blades, Fernanda's eye will sometimes gather light reflecting off the propeller blade and sometimes from the background behind the propeller as the blades move. The rate of movement is faster than the speed at which Fernanda's brain interprets the image, so in her mind she ends up combining information from both the propeller and the background, leading to a sense of a continuous, semi-transparent circle covering the path of the propeller blades.

Question 3: Filter Ordering

[5] marks each.

(a.)

Answer: Anastasia is applying a linear filter (Gaussian smoothing) followed by a non-linear filter (median blur), whereas Sailesh is performing the same filters in reverse order. Non-linear operations are not commutative, so for most images their individual code will lead to different outputs.

(b.)

Answer: Both Anastasia and Sailesh are applying two non-linear filters (median blur) to the image, but the sizes of the kernels is reversed. Non-linear operations are not commutative, so for most images their individual code will lead to different outputs.

(c.)

Answer: NOTE: There was a typo in this question, and Anastasia and Sailesh were actually applying the same filters in the same order, so the answer is trivial. However, had the question been written correctly in the first place and the orders of the filters been reversed, the images would nevertheless have been identical, as the following answer states:

Both Anastasia and Sailesh are applying two linear filters (Gaussian smoothing) to the image, but the sizes of the kernels and standard deviation of the Gaussian functions are reversed in order. Both filters are symmetric (so cross-correlation is equivalent to convolution), and convolution is commutable, so their code will result in identical output.